

<b>Term</b>	First Semester, Year 1
<b>Module</b>	Quantitative Methods
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<b>Office Hours</b>	Send an e-mail to the professor to ask for an appointment. The TA for this course is Alejandro Forcades (E-mail: alejandro.forcades@uab.cat)

## Description

This course will use measure theory to cover the topics of probability theory, random variables, distributions, and sampling.

Slides, class notes, and problem sets can be downloaded from the professor's webpage

## Objective

The course will provide the foundations and tools of Probability and Statistics that will be used in courses dealing with Economics under Uncertainty and Econometrics.

## Outline

### 1. Probability

Random experiments and sample spaces. Composition of experiments and combinatorics. sigma-algebras, events, and measurable spaces. Measure, probability, and probability spaces. Properties of probability measures. Conditional probability. Independent events. Theorem of total probability. Bayes' theorem.

### 2. Measure Theory

Lebesgue measure. Borel and Lebesgue measurable sets. Finite and sigma-finite measures. Lebesgue-Stieltjes measures. Distribution functions. Measurable, Borel measurable, and Lebesgue measurable functions. Properties of measurable functions. Integral with respect to a measure and its properties. Dominated convergence theorem. The Lebesgue and the Lebesgue-Stieltjes integrals and their relation with the Riemann and the Riemann-Stieltjes integrals. Absolute continuity of measures. Radon-Nikodym theorem. Absolutely continuous functions. Product measures. Fubini's theorem.

### 3. Random Variables and Distributions

Random objects and random variables. Probability distributions. Distribution function of a random variable. Discrete random variables. Continuous and absolutely continuous random variables. Density. Random vectors. Marginal distributions. Independent random variables. Generalized conditional probability. Conditional distributions.

### 4. Expectation

Mathematical expectation. Properties of mathematical expectation.  $L^p$  spaces and the Cauchy-Schwarz inequality. Moments and its properties: mean and variance. Chebyshev's inequality. The moment-generating function of a random variable. Product moments and its properties: covariance and correlation coefficient. Mean

and variance of linear combinations of random variables: the mean vector and the covariance matrix. Multivariate moment-generating functions. Conditional expectation. Conditional expectation given a sigma-algebra. The law of iterated expectations. Jensen's inequality.

## 5. Special Distributions

The discrete uniform distribution and the Dirac distribution. The Bernoulli, binomial, Pascal, geometric, and hypergeometric distributions. The multinomial and multivariate hypergeometric distribution. Integration by parts for Lebesgue-Stieltjes integrals. Lebesgue integration by change of variable: polar coordinates. The uniform density. The gamma, exponential, and chi-square distribution. The beta distribution. The normal distribution. The multivariate normal distribution and its properties.

## 6. Functions of Random Variables

The distribution of a function of a random object. The distribution function of a vector-valued function of a random object. The probability function of a function of a discrete random object and the probability function of a discrete function of a random object. The density of a vector-valued function of an absolutely continuous random vector. Characteristic functions, moment-generating functions, and Laplace transforms of functions of random variables.

## 7. Limiting Distributions

Stochastic processes. Filtrations and martingales. Markov processes. Convergence in probability, in mean square, in distribution, and almost sure convergence. Weak convergence of distribution functions and of probability measures. The Poisson distribution as the limit of binomial distributions. The standard normal distribution as the limit of standardized binomial distributions. Weak and strong laws of large numbers. The central limit theorem.

## 8. Sampling

Samples, random samples, and statistics. The distribution of the sample mean. The distribution of the variance of a random sample and the chi-square distribution. The t distribution. The F distribution.

## References

- Ash, R.B., Real analysis and probability, Academic Press.
- Bierens, H.J., Introduction to the mathematical and statistical foundations of econometrics, Cambridge Univ. Press.
- Billingsley, P., Probability and measure, John Wiley.
- DeGroot, M.H., Probability and statistics. Addison-Wesley.
- Hoel, P.G., Introduction to mathematical statistics. John Wiley.
- Hogg, R.V. and Craig, H., Introduction to mathematical statistics. McMillan.
- Lindgren, B.V., Statistical theory. McMillan.

## Grading

Students must solve compulsorily 6 problem sets. There will be a final exam.