Asymmetric internal leadership in confronted parties

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Abstract

In this paper we focus on the effect of party leaders on the total partisan turnout. We analyze a model of two party electoral competition and characterize the optimal number of leaders for each party and examine how may it change with respect to: leadership costs, intensity of the leaders' influence on voters, heterogeneity of voters' preferences, dispersion of the voters' preferences over the ideological space. When we consider identical distributions of partisan preferences for both parties we find that the number of optimal leaders for a party: is always a decreasing function of the leadership costs; may increase with the intensity of their influence and may decrease with the dispersion of the voters' preferences over the ideological space. We also find that parties whose partisans have more moderate preferences enjoy an advantage al high leadership costs. This could become a disadvantage at small enough leadership costs.

Key Words Endogenous leaders, voter mobilization.
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1 Introduction

In any competitive and democratic contend between parties, these institutions try to capture the support of the greatest number of voters. In the pursuit of this objective, defining a political platform is not enough. In addition to that, parties need the active work of citizens who may be closer to the population than the party itself. Leaders have well defined ideal policies, which are essential to determine how many people they are going to convince. This paper deals about parties' optimal selection of the number of leaders.

In this piece of research we are not going to focus on the particular characteristics of these leaders. Becoming a leader implies monetary or sociological benefits, for which citizens may compete. Political ability, charisma and intelligence, among others, are features that can define the identity of the citizens who reach this position. As long as these characteristics are uncorrelated with ideology, parties are not able to control leaders ideology.

Since, from the parties' perspective, leaders are useful tools to obtain votes, it may be interesting to specify those reasons that explain why one of the parties may end up having more leaders than the other. In this paper, we are going to analyze how differences in the population of partisans for each one of the parties implies an equilibrium with different number of leaders for each one of them. We are going to analyze how the size of the population, the size of the ideological spectrum each party is facing, leadership costs or asymmetries in the population's ideological distribution could well explain the asymmetries in the number of optimal leaders for each party.

Once we assume the possibility of influencing, we get into one of the questions that has created greatest discussion in political economics: whether the act of voting is rational. The answer to this question depends on how the act of voting itself and the consequences of such an action can affect the utility of the agent that is actually voting. However, there has been little success in explaining the action of voting whenever the weight of a single vote, and consequently the agents' utility, are tiny.

We may consider the model of Riker and Ordeshook [15] as an initial reference when facing the preceding problem. They first argued about the equation that a rational voter should face when deciding if going to vote or abstaining. An individual will only vote if the benefits from voting, that is, what she gains if her preferred party wins the election weighted by the probability that her vote will change the outcome, are bigger than her costs of going to vote.

However, in many voting procedures, the probability that a single vote might change the result is very small. For instance, in the U.S. the probability that a single vote will affect the final outcome of a presidential election has been estimated to be around 1 in 10 million. Even if we only considered some small states in specific close elections (such as Nevada in 1960 or Alaska in 1972), this probability could only rise to 1 in 1.5 million (Gelman, King and Boscarding [4]).

When considering voting costs and the benefits that an individual may get from voting, one needs to refer to the paradox of not voting. According to this, when there are many voters, people should never vote because one vote is never enough to change the result in a way that is relevant to the voter's utility. However, if everybody acted like this and decided not to go to the polls, one vote would be enough to determine the winner.

The paradox of not voting has been approached in, basically, two different ways. Changing the essence of the vote has been one way to avoid this problem. Instead of considering the vote as a tool that may decide which party is in government (instrumental approach), voting has been analyzed as the final objective of the action, which directly yields benefits to the voters (expressive approach). People use their vote as a way to express themselves and obtain a positive utility from that. Thus, the paradox of non voting automatically disappears. Beyond that this hypothesis has been useful to explain many voting behaviors.

One of the reasons that has been given to explain high participation in large elections under the instrumental approach is the overestimation of tiny probabilities. According to Quattrone and Tversky [14], voters would assign a greater weight to their vote than the one it really has. They also explain how some of the voters are affected by the so called 'voters illusion'. This effect studies how individuals decide to vote, because they are afraid that if they abstained, those who have similar preferences as them would do exactly the same. They assume that their action is having an effect on others' actions, even if it has not.

A different way of facing this problem, following again the instrumental approach, is assuming that only a subset of the population rationalizes the problem stated above. The vote of each one of these citizens in the subset has, still, a tiny weight; but the votes they obtain in the society through their influence is not neglictible. This implies that the rest of voters will partially follow the influencing citizens' advice, or, even more, will imitate them. These models which divide the citizens between leaders and followers have commonly been defined as group-based models.

The number of leaders belonging to each one of the parties is an issue of great importance, because these are the ones that obtain effective votes for their respective party. For this reason, the purpose of this paper is to analyze the different features that may explain a larger number of leaders in society, which may induce a higher participation. We will also investigate the factors that could induce a different number of leaders to different parties. The difference in the number of leaders is vital, because it could sometimes be the only reason to explain a winning party.

Before analyzing the methodologies, perspectives and results that the literature above mentioned has provided, we should ask ourselves if belonging to any specific group does affect someone's decisions. It has been examined how membership to social groups may explain political behavior. Brooks and Manza [2] have shown how variables such as individuals' gender or belonging to an ethnic group have, since 1960, increased their significance explain voters' behavior. The importance of the religion and the social class of individuals has been relatively stable since then. Anyway, the late expansion of global communications do not seem to diminish the effect of these characteristics, cause and effect of the interaction of people, on voters behavior. Belonging to the same social group could imply these individuals having similar preferences and, therefore, behaving similarly. Since such a study is not able to identify whether the cause of a homogenous behavior are the similar preferences or a potential influence, we should not discard any of them. Recently, Nickerson [12] managed to disentangle these effects and prove that there is a great amount of political influence among couples.

The treatment that social sciences have given to the influence that leaders provoke in followers is beyond the fact that group based models avoid the non voting paradox. And, even if ideological closeness is enough reason to explain similar behavior, it should be analyzed how such an important concept in sociology, as it is influence, may affect the result of some elections.

There have been many attempts in order to classify the society with respect to the influence that different groups provoke among them. This classification has been made, in many occasions, following discrete variables. Although limiting the whole population to a discrete, or even a dichotomous choice, may not be very realistic, this is the methodology that has been used by many sociologists. There has been a tendency not to stratify too much the society with respect to personal influence, which allows for a good description of each group.

A good example of that is given by Roper [17] who divides, theoretically, the society between 6 levels of influence and gives a rough estimation about how many US citizens would be in each level. According to him only the last two of them, the Participating Citizens (10 to 25 million), and the Politically Inert group (75 million), include a significant part of the population, large enough as not to be conformed by elected politicians, neither neglectable among the whole population (the numbers given in parentheses correspond to the estimations of people in each group of US citizens in 1954).

The division of citizens in Participating Citizens and the Politically Inert group is quite consistent with the literature dealing with the two-step flow hypothesis. This was started by Lazarsfeld [7] and it has in common with the preceding approach that it also divides citizens in two main categories: opinion givers and opinion receivers. This formulation, indeed, differs somehow from the preceding one, because it assumes that opinion givers influence opinion receivers, while from the perspective of Roper, the members of the last group were not participating in any political discussion.

This literature has been criticized from different perspectives. On one side we may find Robinson [16], who argued in favor of the division of the opinion receivers in different categories with respect to their attitude toward the source of influence. At that time, when media started its particular revolution, Robinson asked for at least considering a different treatment for those who just attended the media and the ones who were actively looking for a more personal source of opinion. Heinz Eulau [3] criticized this approach, by arguing it was social determinism. Moreover, Downs considered that it 'violated the principles of rational choice theory' (Zuckerman [19]). In this paper, we have used similar tools to the ones from the two-step flow hypothesis, to be applied in a completely rational context. So, Lazarsfeld's [7] contributions are not as far from rationality as claimed.

This literature was criticized because of its extreme simplicity, especially when the media developed so that a slightly more complex structure was needed to examine the phenomena. The critique on which this paper is partially based is the already mentioned study made by Robinson [16]. He argues about the different kinds of people that are considered in the group of opinion receivers, especially after the huge development of media at that time. He made the hypothesis of differentiating two kinds of people among those that are opinion receivers: the ones that only attended to media and those who looked for a more personal source of opinion. The latter ones are the ones that we are interested in. They looked for a source of opinion closer to their own one than the one they would receive from the media. Although he did not provide a formal model to explain these effects, he proved the existence of these groups and the flows of information with a survey.

There have also been group-based models in Political Economy, which must not be confused with the citizen candidate models. The basic difference of the group-based models from the rest of the models that also have an instrumental approach is that the first ones move the strategic cost-benefit analysis one step up. Leaders are agents created by political parties who will get a large benefit from the victory of their party. So, they incur in large costs to convince other citizens to vote for this party. The private benefits of leaders should be defined, so that a group based model is not considered a citizen candidate model. Citizen candidate models (Besley and Coate [1]) assume that leaders have political objectives. According to that, they are the ones running for office and, therefore, voters care about the features of the candidates, instead of the parties'. This is not the case in group based models where leaders are trying to get the votes, not for themselves, but for the party.

Throughout the description of the model, given in section 2, it can be noticed that a particular leader may only influence a follower if they agree on the candidate. It is a common feature of these models (Shachar and Nalebuff [18]) to assume that leaders cannot switch followers opinion, but they can convince them to take an action they are satisfied with. This reflects that it is much easier for leaders to mobilize votes than to convince those who think differently.

Unlike many of the models, which take exogenous leaders (shachar and Nalebuff, [18]; Glaeser et al. [6]), the number of leaders in this paper is endogenous. These leaders could be taken as the directors of endogenously formed groups. A condition to establish a group is that its members must have similar views. The creation of these groups will be based on a concept called homophily. Homophily is the natural tendency of individuals to associate with those who think similar to them. This phenomena was analyzed by Precker [13], who, in a study, found that most individuals like others who are similar to them, even before any mutual friendship has been stated. Lazarsfeld and Merton [8] distinguished between status homophily and value homophily. Value homophily is the tendency that individuals have to join those who think similar to them. On the other hand, status homphily means that people who are in the same social status tend to associate among them. However, these two concepts can be correlated when we talk about political views. For the purpose of this analysis, I will only focus on value homophily.

Herrera and Martinelli [10] examined a model, where leaders endogenously arised. They described the equilibrium and some of its features in a circular space. However, some of the essence of the problem of the political competition cannot be analyzed in a circular space. If we consider the traditional left-right division, the two extremes have little in common except their extremism (in any case, much less in common than two centrist voters). Even though the construction of the model in this paper has some elements in common with the one of Herrera and Martinelli [10], its main difference is that we move from the circular space to the one dimensional segment. This transition needs some different treatments and allows for new features to be considered.

A few words are needed about how influence is used in this paper. Influence has been modelled in very different ways. It has been treated as a channel in order to share information. This approach has been taken by Mattozzi [11]. There are other authors who have explored possible equilibria in models with influence, where it allows some individuals to have control over some others. Martinelli and Herrera [10], and Glaeser et al. [6] are good examples of models where followers act exactly in the same way as the leaders who are influencing them.

The model we discuss is an adaptation of the one by Martinelli and Herrera [10]. In their model leaders arise along a circumference, which does not allow to introduce any ideological difference between the two competing parties. We think that representing the political space by the real line is closer to the reality of many political competitions. Not only that, but the location of any agent along the line has now a known meaning that might be useful to apply to reality, that is, ideal policies. We will check how the main features of the equilibria are equivalent in this setup as well. Leaders and followers will arise endogenously while there will always exist some citizens who abstain. These, due to the construction of the model, will always be located in the center.

We have found different arguments that explain why one of the parties has a greater number of leaders than the opponent. We examine how leadership costs or differences in the efficiency among leaders may explain asymmetries in the number of leaders. On the other side, a different size of the political spectrum may yield a completely symmetric equilibrium if parties only maximize the votes they can ensure and there is a perfect influence. Introducing asymmetric distributions of citizens' ideals between parties may also explain these differences.

In section 2, the basics of the model and the definition of equilibria are given. Moreover, we precisely define the equilibrium for the continuous case where parties have voters and leaders uniformly distributed. We characterize the unique equilibrium and prove its existence. Furthermore, we examine how leadership costs and efficiency explain asymmetries in the optimal number of leaders. It is also explained how enlarging the size of the political spectrum of any party may not have any effect into altering the symmetric equilibrium.

In section 3, we examine some of the effects that would arise when assuming asymmetric distributions of citizens' ideals among parties. Extending the analysis to non uniform distributions is too complex and it does not yield any clear theoretical result. However, the model can be a good tool when making simulations with the different distributions. In order to give an intuition, we have examined runs where the potential voters of each party arise from different beta distributions. These results cannot be so easily generalized to any distribution, but show us the incidence that asymmetries in distributions may yield asymmetries in the number of leaders. It is also mentioned along the paper how the insertion of an influence function decreasing in the ideological distance may alter the results.

In section 4, we study a discrete case. The objective is not to analyze an election with few voters, but one where parties have an internal structure. This internal structure is characterized by some previously defined groups, each of them having a positive measure of votes[1]. By extending the leaders followers model to the internal structure of one party, we examine how asymmetric distributions combined with an internal structure may play a role explaining differences in the number of leaders. Two general cases are analyzed. In the first one, the internal structure of one party is biased to the center, while in the second one it is biased to an extreme.

A resume of the results, conclusions and ideas for further research are offered in section 5.

2 The model. Continuous case.

Let us assume a political competition between two parties: A and B.

The ideal policies of the population are distributed in the policy space $[-1, 1]$ as follows. Half of the population is uniformly distributed over the segment $[-1, 0]$ and the other half is uniformly distributed over $[0, 1]$ [2].

Each of the parties (A and B) selects a countable set of leaders. Leaders of party A are randomly distributed over the segment $[-1, 0]$ and those of party B are randomly placed over $[0, 1]$ [3]. Leaders are the connection between parties and voters, and the distribution of their ideals along the

[1]Unlike the continuous case, where leaders' votes as well as individually considered followers' are neglectable.

[2]The population could be obviously distributed uniformly $[-1, 1]$. These distributions are given separately because modifications are done separately as well.

[3]This randomization can be explained by assuming that the ability to become a leader is a function of some abilities which are uncorrelated to ideology

policy space is the same as the one of the voters.

Leaders are parties' tools. The only way parties are able to obtain votes is through leaders. Leaders represent political activists that obtain, through their work and connections, votes for their preferred party. A leader convinces all those citizens that have a more extremist ideal policy to vote. Since leaders form a countable set, their weight in the elections is neglectable and so, we will just focus on the convinced voters.

In order to explain how leaders convince voters, we need first to assume that we have L_A party A leaders and L_B party B leaders. Each of these leaders has an ideal policy which is uniformly distributed along the policy space of their respective party, exactly the same as voters. Take all the leaders of each of the parties and order them with respect to the distance from 0 to their respective ideal policy. Thus, $x_{1,A}$ is the party A's leader who has an ideal policy closest to 0 (the most centrist one) and $x_{L_A,A}$ is the party A's leader who has the ideal policy furthest from 0 (the most extremist one). Then, being $x_{i,A}$ the ideal policy of some leader i, she will convince to vote to all those voters who have an ideal policy in the segment $(x_{i-1,A}, x_{i.A}]$ [4].

The description given above, in order to explain how party A leaders obtain votes for their party, works exactly in the same way for party B (see figure 1), where each arrow starts from the ideology of the leader and covers all the space of those who are being influenced.

The number of leaders is decided by parties which are office motivated. Getting one extra leader implies a fixed cost for the party ($C > 0$). Winning the elections implies a benefit, for the moment, equal for both parties. However, since, by the construction of the model, the parameter of interest to parties is the ratio cost-benefit, the benefit of winning the elections can be normalized to 1.

When parties decide how many leaders they want, they compare the cost of having one extra leader, C, with the extra winning probability that they would obtain with that leader. If the difference in the probability of winning the election is greater than the cost, then parties would want to have more leaders; otherwise, they would not.

In order to characterize the equilibrium, a few definitions are needed. Let L_A and L_B be, respectively, the number of leaders of party A and B.

[4] The voting scheme that results from this assumption is equivalent to one where voters have single peaked preferences, where they are influenced by their preferred leader and where a party only considers the votes that can be ensured. In the extension a different influence is discussed.

Then, $P_j(L_A, L_B) \geq 0$ for $j = A, B$ is the probability that party j has of winning the elections. Since there are only two parties it is obviously the case that $P_A(L_A, L_B) = 1 - P_B(L_A, L_B)$, that is, the sum of both probabilities must be one.

Now, let us define the marginal benefit. The marginal benefit is the increase in the probability of winning carried by the last leader of that party. That is, the difference in the probability of winning compared to the situation where the party had one leader less. Named $MB_j(L_A, L_B)$ the marginal benefit of party j, then:

$$MB_A(L_A, L_B) = P_A(L_A, L_B) - P_A(L_A - 1, L_B)$$
$$MB_B(L_A, L_B) = P_B(L_A, L_B) - P_B(L_A, L_B - 1)$$

Then, an equilibrium should be a pair (L_A, L_B), such that adding a new leader would not carry an increase in the expected benefit greater than C. Equivalently, having one less leader would imply a reduction in cost, compared to the loss suffered in the expected benefit. This conditions are summarized in the following definition of equilibrium.

Definition 1. *A pair (L_A, L_B) is an equilibrium as long as:*

$$MB_A(L_A, L_B) \geq C \text{ and } MB_A(L_A + 1, L_B) < C$$

$$MB_B(L_A, L_B) \geq C \text{ and } MB_B(L_A, L_B + 1) < C$$

2.1 The equilibrium with symmetric uniform distributions.

We will consider the features given in the description of the basic model as the benchmark. In this section we characterize some of the equilibrium conditions of this setup. In the following sections we change some of the hypothesis to analyze how the results obtained here change.

Consider the setup given above and take L_A and L_B leaders for parties A and B respectively, so that we have totally $L_A + L_B$ leaders. Take each one of the parties $j = A, B$ separately and put in order its leaders according to the following rule: for each party $j = A, B$ we will name $x_{1,j}$ the ideal policy of the leader of party j who is nearer from 0; $x_{2,j}$ will be the second one and we will follow this rule so that $x_{L_j,j}$ is the ideal policy of the leader who is the furthest from 0, that is, the most extremist. According to this rule, if we represent the leaders' ideals from the smallest to the biggest one, we have the following series:

$$x_{L_A,A}, x_{L_A-1,A}, ..., x_{1,A}, x_{1,B}, x_{2,B}, ..., x_{L_B,B}$$

We will use order statistics of random variables, because leaders are randomly taken from the distribution of the population. It will be useful to remember that the density function of the k^{th} order statistic ($f_{x_k}(x)$), as a function of the cumulative distribution function and the density function of a random variable x, in a sample of size n, is given by:

$$f_{x_k}(x) = \frac{n!}{(k-1)!(n-k)!} F(x)^{k-1}[1-F(x)]^{n-k} f(x)$$

where $F(x)$ and $f(x)$ are, respectively, the cumulative distribution function and the density function of x.

Note that when computing the number of votes that go to each of the parties, we have only two important elements: $x_{1,A}$ and $x_{1,B}$. The location of the ideal policies of the most centrist leaders of each party are going to determine the total number of votes that will go to each of the parties. However, due to the randomization of their location, the total number of leaders is going to be the variable of importance.

Divide all leaders in two separate groups according to their preferred party and keep them in order with respect to their ideal policies, from the smallest (closest to -1) to the largest one (closest to 1). Thus, we will have two groups: the ideal policies of the leaders of party A[5] and those of B[6]. Take, for each of the two parties, the leaders' ideal policy that is the nearest from 0, these are $x_{1,A}$ and $x_{1,B}$. These two variables are order statistics and we will calculate their density function.

Let us start with the density function of $x_{1,B}$, which is the first element of a group of L_B that is taken from a uniform distribution. Then its density function is given by $f_{x_{1,B}}(x_B) = L_B(1-x_B)^{L_B-1}$, defined over the segment $[0,1]$.

On the other hand, $x_{1,A}$ is the last element of the group of L_B leaders of party B, where all of them are uniformly distributed. Then, the density function of $x_{1,A}$ is given by $f_{x_{1,A}}(x_{1,A}) = L_A(x_{1,A})^{L_A-1}$. In order to make the computations easier, we will use a transformed variable $z_{1,A} = 1 + x_{1,A}$. This transformation makes the calculus due to two reasons: first, $z_{1,A}$ has the same density function as $x_{1,A}$ ($f_{x_{1,A}}(x_{1,A}) = L_A(x_{1,A})^{L_A-1}$); second, $z_{1,A}$ is defined in the segment $[0,1]$, the one over we have defined the density function of $x_{1,B}$.

[5]with ideals $(x_{L_A,A}, x_{L_A-1,A}, ...x_{2,A}, x_{1,A})$.

[6]with ideals $((x_{1,B}, x_{2,B}, ..., x_{L_B-1,L_B}, x_{L_B,B})$.

Since these two random variables are independent, we can easily define their joint density function:

$$f_{z_{1,A},x_{1,B}}(z_{1,A},x_{1,B}) = L_A L_B (z_{1,A})^{L_A-1}(1-x_{1,B})^{L_B-1}$$

Now, we will analyze the probability that party A has of winning. This is the probability of the length of the segment $[-1, x_{1,A}]$ being greater than the one of $[x_{1,B}, 1]$. That is, the probability that $[x_{1,A} - (-1)]$ is greater than $[1 - x_{1,B}]$. Or, equivalently, the probability that $z_{1,A}$ is greater than $[1 - x_{1,B}]$. Therefore:

$$P_A(L_A, L_B) = \int_0^1 \int_{1-x_{1,B}}^1 L_A L_B z_{1,A}^{L_A-1}(1-x_{1,B})^{L_B-1} dz_{1,A} dx_{1,B}$$

$$= \int_0^1 L_B(1-x_{1,B})^{L_B-1} dx_{1,B} - \int_0^1 L_B(1-x_{1,B})^{L_A+L_B-1}$$

$$= -1(-1) + \frac{L_B}{L_A+L_B}(-1) = 1 - \frac{L_B}{L_A+L_B} = \frac{L_A}{L_A+L_B}$$

Then, it is obviously the case that $P_B(L_A, L_B) = \frac{L_B}{L_A+L_B}$.

Now, that we have $P_j(L_A, L_B)$ for $j = A, B$, we can compute $MB_j(L_A, L_B)$.

$$MB_A(L_A, L_B) = P(L_A, L_B) - P(L_A - 1, L_B) = \frac{L_A}{L_A+L_B} - \frac{L_A - 1}{L_A+L_B-1}$$

$$MB_A(L_A, L_B) = \frac{L_B}{(L_A+L_B)(L_A+L_B-1)}$$

Lemma 1. *In the symmetric case, the marginal benefit of a party decreases with the number of leaders of that party (i.e.* $\frac{\partial MB_j}{\partial L_j} < 0$ *for* $j = A, B$*).*

(Note: all proofs can be found in the Appendix).

Lemma 2. *In the symmetric case, the marginal benefit of the party with more leaders increases with the number of leaders of the opposite parties (if* $L_j \geq L_k$*, then* $\frac{\partial MB_j}{\partial L_k} \geq 0$*).*

(See Appendix).

The intuition of Lemma 2 is the following. Assume we are analyzing the decision of party A and, in that situation, it has many more leaders than party B. Then, even if it is quite sure that party A is going to win the election, it would almost win it anyway without one of those leaders. However, if the number of leaders in party B is bigger and bigger, so that it comes closer to the number of leaders in party A, losing one leader becomes a more important issue for party A.

2.1.1 Existence and uniqueness of equilibrium.

Since we want to characterize completely the equilibrium, we should first notice that the probabilities and marginal benefits we have calculated up to this point are for positive values of L_A and L_B. Then, let us analyze all the possible cases.

First of all, may $(0, 0)$ be equilibrium? In order to be so, deviations cannot be optimal. Going from such situation to one where one of the parties had only one leader, would imply improving the probability of winning from $\frac{1}{2}$ (which is the probability of winning for each party when there are no leaders) to 1 for the party that has one leader. When there are no leaders nobody votes, and since both parties draw, each of them wins with probability $\frac{1}{2}$. Hence, $(0, 0)$ is an equilibrium as long as $\frac{1}{2} < C$.

Lemma 3. *In the symmetric case, a pair of strategies $(L + j, L)$ for $j > 0$ can never be an equilibrium.*

(See Appendix).

So, if the equilibrium exists, it must have the form $(L^\star, L^\star)$. In that case, the marginal benefit for each party would be $MB_j(L^\star, L^\star) = \frac{1}{2(2L^\star-1)}$. The marginal benefit is strictly decreasing in L. It goes to zero when L goes to infinity and when $L = 1$, $MB_j = \frac{1}{2}$. Having described this features, note that there will always be equilibrium. It will be $(0, 0)$ if $C > \frac{1}{2}$. Otherwise, take the biggest L that makes $\frac{1}{2(2L-1)} \geq C$ hold. That will be an equilibrium because the marginal benefit is strictly decreasing and tends to zero.

Proposition 1. *In the symmetric case the equilibrium exists and is unique. If $C > \frac{1}{2}$, then the equilibrium is $(0, 0)$. On the other side, if $C \leq \frac{1}{2}$, then the equilibrium is having $L^\star$ leaders in each party, where $L^\star$ is the biggest L for which the inequality $\frac{1}{2(2L-1)} \geq C$ holds.*

This equilibrium is invariant with respect to the size of parties' political spectrum. In order to check this, assume that the ideals of the potential voters of party B are uniformly distributed over the segment $[0, \alpha]$, where $\alpha > 1$ and everything else remains as before. Since the marginal benefits are exactly the same, all the results continue to hold.

Proposition 2. *In the symmetric case, increasing the political spectrum to be covered by one party has no effect in the equilibrium.*

(See Appendix).

2.1.2 Cost asymmetries.

Let parties A and B have different costs in order to create a leader. There might be many reasons that could explain such difference in costs; for instance, historical reasons that make the participation of leaders more usual in one party or the interaction of the political party with other institutions, such as the religious ones.

Lemma 4. *If one of the parties has smaller leadership costs than the other ($C_i < C_j$), then it will have weakly more leaders than the other party ($L_i \geq L_j$).*

(See Appendix).

The conclusion of this proposition is very clear and intuitive. Whenever one of the parties has smaller (marginal) costs to obtain more leaders than the other party, it will have no less leaders in equilibrium.

2.1.3 Electorates with different size.

Now, let us consider an asymmetric situation, where party B suffers from a disadvantage. It could be that due to bad communications channels or to their lower leadership skills, the leaders of one party would only obtain a fraction of votes from the people they influence. We will maintain all the original assumptions, but party B is only going to receive a fraction $\frac{1}{\beta}$ of the votes, where $\beta > 1$[7]. Then, the joint density function of $(z_{1,A}, x_{1,B})$ would not change. But the probability for party A winning the elections would be equal to the probability of having $z_{1,A} \geq \frac{1-x_{1,B}}{\beta}$. We could calculate this using the following integral:

$$\int_0^1 \int_{\frac{1-x_{1,B}}{\beta}}^1 L_A L_B z_{1,A}^{L_A-1}(1-x_{1,B})^{L_B-1} dz_{1,A} dx_{1,B}$$

$$= \int_0^1 L_B(1-x_{1,B})^{L_B-1} dx_{1,B} - \int_0^1 \frac{L_B(1-x_{1,B})^{L_A+L_B-1}}{\beta^{L_A}} dx_{1,B} = 1 - \frac{L_B}{(L_A+L_B)\beta^{L_A}}$$

[7]This would be equivalent to assume that party A has a larger electorate.

From here, we can easily compute party A's marginal benefit:

$$MB_A(L_A, L_B) = 1 - \frac{L_B}{(L_A + L_B)\beta^{L_A}} - 1 + \frac{L_B}{(L_A + L_B - 1)\beta^{L_A - 1}}$$
$$= \frac{L_B[1 + (\beta - 1)L_A + (\beta - 1)L_B]}{(L_A + L_B)(L_A + L_B - 1)\beta^{L_A}}$$

As well as the one for party B:

$$MB_B(L_A, L_B) = \frac{L_B}{(L_A + L_B)\beta^{L_A}} - \frac{L_B - 1}{(L_A + L_B - 1)\beta^{L_A}}$$
$$= \frac{L_A}{(L_A + L_B)(L_A + L_B - 1)\beta^{L_A}}$$

This tools are indeed very useful to justify the following proposition, which could be named the 'crushing' effect:

Proposition 3. *In the symmetric case, if leaders from party i obtain a higher proportion of votes from their area of influence than those of party j, then party i would have weakly more leaders than party j ($L_i \geq L_j$).*

(See Appendix)

It is easy to see that whenever the leaders of one party are more efficient due to, for instance, better communication skills, these leaders are going to have a greater marginal benefit. Therefore, whenever we are in a situation when both parties have equal number of leaders, the party with the most efficient leaders is having a greater incentive to obtain additional leaders. This phenomenon, combined with Lemma 2, makes impossible the existence of any equilibrium where the party with the most efficient leaders has less of them.

Corollary 1. *In the symmetric case, if leaders from party i obtain a higher proportion of votes from their area of influence than those of party j, then party i is expected to win the elections.*

In order to understand this, we ought to calculate party As expected winning margin. One way to do it is by calculating:

$$\int_0^1 \int_0^1 (x_A - \frac{x_B}{\beta}) L_A L_B x_A^{L_A - 1} x_B^{L_B - 1} dx_B dx_A$$
$$= \frac{\beta L_A(L_B + 1) - L_B(L_A + 1)}{(L_A + 1)(L_B + 1)} = \frac{L_A L_B(\beta - 1) + L_A\beta - L_B}{(L_A + 1)(L_B + 1)}$$

which is strictly greater than zero when $L_A \geq L_B$.

2.2 The effect of a decreasing influence function with a different size of the ideological spectrum.

Up to now we have been assuming that each leader i of party A (resp. B) would convince all those individuals with an ideal between the one of the leader and the next most extremist leaders; that is, all those having an ideal in the segment $(x_{i+1,A}, x_{i,A}]$ (resp. $[x_{i,B}, x_{i+1,B})$). The influence that a leader voting for party j with ideal $x_{i,j}$ has over a citizen with an ideal x is named $\theta(x_{i,j}, x) = \max\{0, 1-\lambda|x_{i+1,j} - x_{i,j}|\}$, where $\lambda > 0$ is a parameter that measures how influence decreases with distance. θ is assumed to be nonnegative, because we have no role for negative influence.

Dealing with general distribution functions and a decreasing influence function makes the problem much more difficult, because we are adding as many discontinuities as leaders. However, we can give the intuition for the general uniform case where the ideals of the individuals ready to vote for one party follow a distribution $U(0, \alpha)$. We are interested into calculating the expected distance between the ideals of two consecutive leaders, $(x_{i+1,j} - x_{i,j})$; that is, the expected area of influence for leader i of party j. The joint density function of $(x_{i,j}, x_{i+1,j})$ is given by:

$$f(x_{i,j}, x_{i+1,j}) = \frac{L_j!(\frac{x_{i,j}}{\alpha})^{i-1}(1-\frac{x_{i+1,j}}{\alpha})^{L_j-i-1}}{\alpha^2(i-1)!(L_j-i-1)!}$$

If we compute the expected area of influence, $E[|x_{i+1,j} - x_{i,j}|] = \frac{\alpha}{L_j+1}$ (**see Appendix**), we observe that it is increasing in α and decreasing in the number of leaders. Since the influence of each leader is decreasing in the distance, the party with the higher expected areas of influence will be less efficient obtaining votes. Therefore, the party which has the most concentrated population (smaller α) will be the most efficient under a decreasing influence function. Thus, such party will have no less leaders than its competitor.

Remember how we have seen that the length of the uniform distribution did not have any effect on the total number of leaders of any of the parties. However when we introduce a decreasing influence function the most concentrated population increases the efficiency of influence and may have a greater number of leaders at equilibrium.

3 Discrete case: parties with an internal structure.

Extending the previous results to general distributions yields not tractable probability functions, with which general results cannot be stated. In order to examine a case with asymmetric distribution, we are going to analyze a discrete case with few players (potential leaders and followers). The three players in each party represent three groups, ideologically differenced, inside the party. The size of each of the three groups represents both their respective probability of having a leader and their voting size when influenced.

From here in advance we are going to treat these groups as individuals, assuming that there is a complete coordination inside each one of the groups. We can consider that when one citizen in the group becomes a leader, she automatically obtains the support of the group or, equivalently, we can consider that the whole group becomes the leader. The way one group is going to influence the rest is the same as in the continuous case. Therefore, the exact location of the ideals of voters does not affect the result. We will only care about the fact that for each party there is one group which is closer to zero (that may influence the other two), one which is in the middle (that may only influence the most extremist one), and one which has an ideal closer to the extreme (that cannot have an influence on other groups).

We will be assuming perfect coordination inside each of the groups, so that if there is already one leader in the group no more leaders will arise from it. In party A the probability for a leader emerging from any of the groups is equal. For party B groups the probabilities are different. The probabilities of becoming a leader for a citizen that belongs to the most centrist, the intermediate or to the most extremist group are, respectively, r, s and t. Note that $r+s+t=1$. If there are only two leaders to be elected the probability of choosing one of them is equal to its original probability (say r, s or t) over the sum of probabilities of the voters who are not leaders yet. That is, if the first leader of B is the most extremist one (the one that had probability t), the probability that the second is the most centrist one is $\frac{r}{r+s}$ [8]. In the following subsections we will make different assumptions about these parameters to see which are the possible equilibria.

[8] This will be used in order to compute ex-ante probabilities, because there is no way to know if the first leader is the most centrist one or not. Note that the assumed probability satisfies Luce's [9] Choice Axiom.

3.1 A structure very biased to the center.

For the very first case we will be assuming that $r > s > t$, so that party B is biased to the center, and that $s + t < \frac{1}{3}$. Then, the voting size of the two most extremist groups in B are smaller than any group in A. That is, whenever we have a leader in s or in t, its extra voting size (as it is always smaller than $\frac{1}{3}$), will never change the result.

In the following table we show the winning probabilities of party A. In the rows we have the number of leaders of party A, while those of party B are given in the columns.

	0	1	2	3
0	$\frac{1}{2}$	0	0	0
1	1	$1-\frac{5r}{6}$	$\frac{1}{6}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$	$\frac{1}{6}$
2	1	$1-\frac{4r}{6}$	$\frac{2}{6}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$	$\frac{2}{6}$
3	1	$1-\frac{3r}{6}$	$\frac{3}{6}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$	$\frac{3}{6}$

We can, therefore, define the marginal benefit for both parties, the same as we did in the previous section:

	Party A	Party B
$(0,0)$	-	-
$(1,0)$	$\frac{1}{2}$	-
$(2,0)$	0	-
$(3,0)$	0	-
$(0,1)$	-	$\frac{1}{2}$
$(1,1)$	$1-\frac{5r}{6}$	$\frac{5r}{6}$
$(2,1)$	$\frac{r}{6}$	$\frac{4r}{6}$
$(3,1)$	$\frac{r}{6}$	$\frac{3r}{6}$
$(0,2)$	-	0
$(1,2)$	$\frac{1}{6}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$	$(1-r)\frac{5}{6}-\frac{st}{2(r+t)}-\frac{st}{2(r+s)}$
$(2,2)$	$\frac{1}{6}$	$(1-r)\frac{4}{6}-\frac{st}{2(r+t)}-\frac{st}{2(r+s)}$
$(3,2)$	$\frac{1}{6}$	$(1-r)\frac{3}{6}-\frac{st}{2(r+t)}-\frac{st}{2(r+s)}$
$(0,3)$	-	0
$(1,3)$	$\frac{1}{6}$	$\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$
$(2,3)$	$\frac{1}{6}$	$\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$
$(3,3)$	$\frac{1}{6}$	$\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$

First of all, we will characterize which are the possible equilibria when the groups of party B are very biased to the center. We will represent this by assuming that $r > s > t$. As follows, we will analyze the results of the next table which contains the set of possible equilibria (the number of party A leaders are given in rows and those of party B in columns).

	0	1	2	3
0	Yes	Yes	No	No
1	No	Yes	Yes	No
2	No	No	No	No
3	No	Yes	Yes	Yes

3.1.1 Same number of leaders.

We will start by characterizing all the possible equilibria where both players have the same number of leaders. $(0,0)$ is a special case because the possible deviation of any of the two parties (i.e. acquiring one leader) would imply for that party to obtain the highest possible marginal probability. If one party deviates, its winning probability changes from a fair coin toss to a sure victory. Then, the marginal probability of the party that deviates is $\frac{1}{2}$. So, whenever $C > \frac{1}{2}$, $(0,0)$ is an equilibrium, and since there are no more locations where both parties obtain such a large marginal probability, $(0,0)$ will be the only equilibrium in this case.

$(1,1)$ and $(3,3)$ can be equilibria. Note that we are not ensuring the existence for any distribution of $\{r,s,t\}$. Instead of that we are ensuring that there exist some values of $\{r,s,t\}$ and C, for which these equilibria may exist **(See Appendix)**.

$(2,2)$ can never be an equilibria, because $MB_A(3,2) = MB_A(2,2)$. That is, whenever party B has two leaders, the extra benefit that the third leader would obtain in A is the same as the one that obtains the second one. Note as well that $(1,1)$ and $(3,3)$ cannot be equilibria at the same time, because $MB_B(1,2) > MB_B(3,3)$. Not only that but, the smaller the cost the greater is going to be the optimal number of leaders **(See Appendix)**.

3.1.2 More leaders for party A.

We will analyze all the possible pairs of leaders where party A has more leaders than B to determine which of them can and which cannot be equilibrium. $(1,0)$ cannot be an equilibrium. In order to be so it would be necessary that $MB_A(1,0) = \frac{1}{2} \geq C$ and $MB_B(1,1) = \frac{5r}{6} < C$. However, since $r > \frac{2}{3}$, then $\frac{5r}{6} > \frac{5}{9}$.

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$(3,1)$ can be an equilibrium for values of r sufficiently close to 1. $(3,2)$ can be an equilibrium as well **(See Appendix)**.

$(2,0)$ and $(3,0)$ cannot be equilibria, because at any of them the marginal probability for party A is 0 and $C > 0$.

$(2,1)$ cannot be an equilibrium because $MB_A(3,1) = MB_A(2,1)$. These equilibria cannot coexist. Similarly as before, a smaller C would imply a greater number of leaders **(See Appendix)**.

3.1.3 More leaders for party B.

With the purpose of analyzing all the possible equilibria we will end by looking for these in every pair of leaders with more leaders for party B than for A. Among these, we can nd that $(0,1)$ is an equilibrium as long as $MB_B(0,1) = \frac{1}{2} \geq C$ and $MB_A(1,1) = 1 - \frac{5r}{6} < C$ **(See Appendix)**.

$(1,2)$ can be an equilibrium **(See Appendix)**.

By the same argument given in the preceeding subsection $(0,2)$ and $(0,3)$ never are equilibria. In addition, $(1,3)$ and $(2,3)$ cannot either be equilibria **(See Appendix)**.

$(0,1)$ and $(1,2)$ cannot coexist and $(1,2)$ is an equilibrium for smaller values of C than $(0,1)$ **(See Appendix)**.

3.1.4 Uniqueness the equilibrium.

Lemma 5. *In the discrete case, when one of the parties has a uniform structure and the other one is very biased to the center, if the equilibrium exists, it is unique.*

In each of the subsections we have seen that none of the equilibria may coexist with others belonging to the same group. For the rest of pairwise comparisons, see the appendix.

We have characterize many different equilibria that can be distributed in three groups. The first group is conformed by all those equilibria where both parties have the same number of leaders. In $(0,0)$, where nobody votes, and in $(3,3)$, where everybody does, the two parties are going to obtain the same quantity of votes. However, the result in $(1,1)$ is not symmetric. Although both parties have one leader, the one from B is more effective, yielding a greater probability of winning.

In the second group we can find those equilibria where party B has the most leaders. In any of these party B has an advantage. In $(1,2)$ it has a greater probability of winning than party A, while in $(0,1)$ it ensures the victory.

The third group is conformed by those pairs, where party A has the greatest number of leaders. These are $(3,1)$ and $(3,2)$. There are two important features about these equilibria. First of all, in any of them party A has a greater probability of victory, because it is ensuring all the votes, while this is not the case for party B. Second, since $MB_A(1,1) > \max\{MB_A(3,1), MB_A(3,2)\}$ and $MB_A(2,2) > \max\{MB_A(3,1), MB_A(3,2)\}$, we can ensure that any equilibria where party A has more leaders must hold for a lower value of C than those for which the equilibria with the most leaders for party B hold.

According to the last result, if one of the parties is very biased to the center. It will have an advantage for large enough values of C (as long as they are not larger than $\frac{1}{2}$). However, if C is small (and not too small so that both parties want to have all their leaders), the relatively extremist party could obtain an advantage.

3.2 A structure (not so much) biased to the center.

Now, we will check what happens when party B has an internal structure biased to the center, but where $s+t > \frac{1}{3}$. In this case, if party B has only one leader and it happens to be the intermediate one, this would obtain a higher share of votes than the most extremist leader of A. The winning probabilities for party A are given by:

	0	1	2	3
0	$\frac{1}{2}$	0	0	0
1	1	$\frac{r}{6}+\frac{2s}{3}+t$	$\frac{1}{6}+\frac{st}{3(r+t)}+\frac{st}{3(r+s)}$	$\frac{1}{6}$
2	1	$\frac{r}{3}+s+t$	$\frac{1}{3}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$	$\frac{1}{3}$
3	1	$\frac{r}{2}+s+t$	$\frac{1}{2}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$	$\frac{1}{2}$

We can, therefore, define the marginal benefit for both parties, the same as we did in the previous section:

	Party A	Party B
$(0,0)$	-	-
$(1,0)$	$\frac{1}{2}$	-
$(2,0)$	0	-
$(3,0)$	0	-
$(0,1)$	-	$\frac{1}{2}$
$(1,1)$	$\frac{r}{6}+\frac{2s}{3}+t$	$\frac{5r}{6}+\frac{s}{3}$
$(2,1)$	$\frac{r}{6}+\frac{s}{3}$	$\frac{2r}{3}$
$(3,1)$	$\frac{r}{6}$	$\frac{r}{2}$
$(0,2)$	-	0
$(1,2)$	$\frac{1}{6}+\frac{st}{3(r+t)}+\frac{st}{3(r+s)}$	$\frac{s}{2}+\frac{5t}{6}-\frac{st}{3(r+t)}-\frac{st}{3(r+s)}$
$(2,2)$	$\frac{1}{6}+\frac{st}{6(r+t)}+\frac{st}{6(r+s)}$	$\frac{2s}{3}+\frac{2t}{3}-\frac{st}{2(r+t)}-\frac{st}{2(r+s)}$
$(3,2)$	$\frac{1}{6}$	$\frac{s}{2}+\frac{t}{2}-\frac{st}{2(r+t)}-\frac{st}{2(r+s)}$
$(0,3)$	-	0
$(1,3)$	$\frac{1}{6}$	$\frac{st}{3(r+t)}+\frac{st}{3(r+s)}$
$(2,3)$	$\frac{1}{6}$	$\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$
$(3,3)$	$\frac{1}{6}$	$\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$

We offer a table with the set of possible equilibria:

	0	1	2	3
0	Yes	Yes	No	No
1	Yes	Yes	Yes	No
2	No	No	Yes	No
3	No	Yes	Yes	Yes

3.2.1 Same number of leaders.

We will start by characterizing all the possible equilibria when both players have the same number of leaders. $(0,0)$ is the unique equilibrium whenever $C > \frac{1}{2}$, for the same reasons given in the previous subsection.

$(1,1)$, $(2,2)$ and $(3,3)$ can be equilibria, while non of these equilibria can coexist for the same value of C **(See Appendix)**. Since these equilibria cannot hold at the same time and $MB_A(1,1) > MB_A(2,2) > MB_A(3,3)$, whenever we have an equilibrium with the same number of leader, the total amount of them is decreasing in C.

3.2.2 More leaders for party A.

Let's now examine all the possible equilibria where A party has more leaders than B. With this approach $(1,0)$, $(2,1)$, $(3,1)$ and $(3,2)$ can be equilibrium **(See Appendix)**.

$(2,0)$ and $(3,0)$ obviously cannot be equlibria because the marginal probability of the party A is 0 in any of them and, so, cannot be greater than C.

3.2.3 More leaders for party B.

$(0,1)$ and $(1,2)$ can be equilibria **(See Appendix)**.

$(1,3)$ and $(2,3)$ cannot be equilibria, because $MB_A(1,3) = MB_A(2,3) = MB_A(3,3) = \frac{1}{6}$.

3.2.4 Uniqueness of the equilibrium.

Lemma 6. *In the discrete case, when one of the parties has a uniform structure and the other one biased to the center, if the equilibrium exists it is unique.*

(See Appendix)

In this section as well, we can distribute all the possible equilibria in 3 groups. In the first one we may find all those equilibria where both parties are having the same number of leaders. Again, for obvious reasons, we will leave aside $(0,0)$ and $(3,3)$. The result in $(1,1)$ is not clear, although the party B has the greatest probability of obtaining its most centrist leader, party A has a greater probability than B to obtain an intermediate one. If $5r + 2s > 3$ (i.e. party B is sufficiently biased to the center), then party B will have a greater probability of winning, while the advantage is going to be for party A otherwise. If we are in $(2,2)$, party B is the one with the greater probability of winning.

The second group is conformed by those equilibria where party B has more leaders. Those are $(0,1)$ and $(1,2)$. Whenever $(0,1)$ is an equilibrium, we have that $5r + 2s > 3$, and party B obtains a sure victory. For this to hold it is a sufficiently requirement that $r > \frac{1}{2}$ (B is sufficiently centrist). If the equilibrium is $(1,2)$, again party B is going to have a greater probability of winning ($P_A(1,2) < 12$).

The third group, the one containing the equilibria where party A has more leaders is more heterogenous. On the one side we have $(1,0)$, as long as $5r + 2s < 3$ (i.e. B is not sufficiently centrist). This is also the case in $(2,1)$, where $P_A(2,1) \geq \frac{2}{3}$. Finally, it is obvious that in $(3,1)$ and $(3,2)$ party A has a greater probability of winning. Note that the advantage of party A does not only come from having more leaders, but also from being globally more centrist.

3.3 A structure (not so much) biased to the extreme.

In order not to extend too much the analysis, the results regarding party B to be very biased to the extreme are omitted ($r < s < t$ and $t > \frac{2}{3}$). In that case, if t was very large ($t > \frac{3}{4}$), then the results would be very similar as to considering that party B is centrist biased. This is because, such a large group implies a high coordination (similar to having a large probability of having a centrist voter who obtains the vote for all). If $t < \frac{3}{4}$, then party A would have the first leader for high leadership costs. The extremist biased party would have more leaders for lower values of C.

Therefore, qualitative results remain the same. The extremist biased party would have more leaders for small C, while the relatively centrist one would have more leaders for large C. It would also be true, for any of the cases analyzedLet's consider now that $r < s < t$ and $t < \frac{2}{3}$. The winning probabilities for party A are given in the following table:

	0	1	2	3
0	$\frac{1}{2}$	0	0	0
1	1	$\frac{r}{6} + \frac{s}{3} + \frac{2t}{3}$	$\frac{1}{6} + \frac{st}{6(r+t)} + \frac{st}{6(r+s)}$	$\frac{1}{6}$
2	1	$\frac{r}{3} + \frac{2s}{3} + \frac{2t}{3}$	$\frac{1}{3} + \frac{st}{3(r+t)} + \frac{st}{3(r+s)}$	$\frac{1}{3}$
3	1	$\frac{r}{2} + s + t$	$\frac{1}{2} + \frac{st}{2(r+t)} + \frac{st}{2(r+s)}$	$\frac{1}{2}$

And the marginal benefits for each of the two parties are given by:

	Party A	Party B
$(0,0)$	-	-
$(1,0)$	$\frac{1}{2}$	-
$(2,0)$	0	-
$(3,0)$	0	-
$(0,1)$	-	$\frac{1}{2}$
$(1,1)$	$\frac{r}{6}+\frac{s}{3}+\frac{2t}{3}$	$\frac{5r}{6}+\frac{2s}{3}+\frac{t}{3}$
$(2,1)$	$\frac{r}{6}+\frac{s}{3}$	$\frac{2r}{3}+\frac{s}{3}+\frac{t}{3}$
$(3,1)$	$\frac{r}{2}+\frac{s}{3}$	$\frac{r}{2}$
$(0,2)$	-	0
$(1,2)$	$\frac{1}{6}+\frac{st}{6(r+t)}+\frac{st}{6(r+s)}$	$\frac{5s}{6}+\frac{t}{2}-\frac{st}{6(r+t)}-\frac{st}{6(r+s)}$
$(2,2)$	$\frac{1}{6}+\frac{st}{6(r+t)}+\frac{st}{6(r+s)}$	$\frac{2s}{3}+\frac{t}{3}-\frac{st}{3(r+t)}-\frac{st}{3(r+s)}$
$(3,2)$	$\frac{1}{6}+\frac{st}{6(r+t)}+\frac{st}{6(r+s)}$	$\frac{s}{2}+\frac{t}{2}+\frac{st}{2(r+t)}+\frac{st}{2(r+s)}$
$(0,3)$	-	0
$(1,3)$	$\frac{1}{6}$	$\frac{st}{6(r+t)}+\frac{st}{6(r+s)}$
$(2,3)$	$\frac{1}{6}$	$\frac{st}{3(r+t)}+\frac{st}{3(r+s)}$
$(3,3)$	$\frac{1}{6}$	$\frac{st}{6(r+t)}+\frac{st}{6(r+s)}$

3.3.1 Possible equilibria

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As follows, we have a table with the set of the possible equilibria summarized:

	0	1	2	3
0	Yes	No	No	No
1	Yes	Yes	No	No
2	No	No	No	No
3	No	No	Yes	Yes

Since the explanation about the possible set of equilibria is shorter than in the previous cases, it will be given in one subsection. As always $(0,0)$ is an equilibrium if and only if $C > \frac{1}{2}$. Not only that, but also $(1,1)$ and $(3,3)$ can be equilibria **(See Appendix)**. However, since $MB_B(3,2) = MB_B(2,2)$, $(2,2)$ can never be an equilibrium.

$(1,0)$ and $(3,2)$ can also be equilibria **(See Appendix)**. For obvious reasons, that cannot be the case of $(2,0)$ and $(3,0)$. Moreover, $(2,1)$ can never be an equilibrium, because $MB_A(3,1) > MB_A(2,1)$. Similarly, $(3,1)$ cannot be an equilibrium, since $MB_B(3,2) > MB_A(3,1)$.

$(0,1)$ can be an equilibrium **(See Appendix)**. There are not more equilibria where party B (the more extremist) is receiving the support of a greater number of leaders. For $(0,2)$ and $(0,3)$, it is straightforward. $(1,2)$ is never an equilibria, because $MB_A(2,2) = MB_A(1,2)$. Finally, $(1,3)$ and $(2,3)$ cannot be equilibria because $MB_A(1,3) = MB_A(2,3) = \frac{r}{6} < MB_A(3,3)$.

Lemma 7. *In the discrete case, when one of the parties has a uniform structure and the other one is a bit biased to the extreme, if the equilibrium exists it is unique.*

(See Appendix)

As in the previous subsection, having one structure where one of the parties is a bit biased to the extreme is not enough to determine which party has the first leader most centrist. If $s+3t > 2$, then $(1,0)$ can be an equilibrium, while $(0,1)$ cannot. For that case, whenever we are at $(1,1)$, party A has a greater probability of winning the elections. However, if $s+3t < 2$, the results are exactly the opposite. That would be the only case where party B has more leaders. Otherwise, either we reach to both parties having the same number of leaders or there is non existence in pure strategies.

4 Asymmetric continuous distributions.

Extending the results from section 2 to non-uniform distributions leads to formulas not sufficiently tractable. Anyway, it was needed to examine if asymmetric continuous distributions would be enough to justify a different number of leaders among parties. In order to check this, we could calculate the vote share that the most centrist leader would obtain for a variety of beta distributions. In this section, we consider a distribution $Beta(\alpha, \beta)$ for one party and check how the votes obtained by the expected most centrist leader change with the different values of α and β.

First of all, we should consider what happens when one party decides to run with one leader (see figure 6). Then, the expected location of this leader is obviously given by the mean of the distribution. The mean of a $Beta(\alpha, \beta)$ distribution is $\frac{\alpha}{\alpha+\beta}$. Therefore we should compare the mean with the median of the population to discover which values of α and β yield the greater vote share.

Whenever $\alpha = \beta$, the $Beta(\alpha, \beta)$ distribution is symmetric with respect to its mean. Therefore, the value of the median is exactly that of the mean, and, according to this model, the vote share obtained by this expected unique leader would be half of the potential voters of the party. Then, as long as both parties have, respectively, symmetric populations, the vote share of the expected first leader would be the same. Not only that, but since the size of the ideological spectrum is equal for both sides ($[-1,0]$ and $[0,1]$), the distance, from the ideal policy of this expected first leader to the center, will be the same for both parties. Hence, even if we consider influence toward centrist followers, this result would not change.

This is not the case when $\alpha \neq \beta$, because the location of the median changes with respect to the one of the mean. If $\alpha > \beta$ (resp. $\alpha < \beta$), then the mean is smaller (resp. greater) than the median and, therefore, the vote share obtained by the expected leader is greater (smaller) than $\frac{1}{2}$. Hence, if the potential voters of one party follow a beta distribution where $\alpha > \beta$, while the ones of the other party follow a beta distribution where $\alpha \leq \beta$, then the first party will obtain a greater (marginal) benefit from its first expected leader. Moreover, if one of the parameters is equal for both distributions, the party with the greater difference $\alpha - \beta$, will be the one enjoying the greater vote share from its expected first leader. In any of the two cases, this advantaged party is obviously ready to obtain its first leader for greater leadership costs (C).

Note that the advantaged party is the one for which leaders are expected to be more centrist than half of the populations. This is what we call that one party is more biased to the

We have also analyzed the marginal vote share obtain by the expected most centrist leader, when assuming two leaders (see figure 3) in a population that comes from the distribution $Beta(\alpha, \beta)$. The ideal point of this leader is:

$$E[x_{1,j}(L_j = 2)] = \frac{2}{B(\alpha,\beta)} \int_0^1 x^{\alpha}(1-x)^{\beta-1} B_x(\alpha,\beta)dx =$$

$$= \frac{2\alpha B(\alpha,\beta)}{\alpha+\beta} - \frac{2\Gamma(1+2\alpha)\Gamma(\alpha+\beta)_3F_2(\alpha, 1+2\alpha, 1-\beta; 1+\alpha, 1+2\alpha+\beta; 1)}{\Gamma(1+\alpha)\Gamma(1+2\alpha+\beta)}$$

If we compute the marginal vote share obtained by the expected second leader (see figure 7), we can arrive to more conclusions. First of all, having $\alpha = \beta$ no longer ensures the same expected marginal benefit for the second leader. As we have said, in this case the distributions of the populations of each party are symmetric. While these parameters increase, the populations of each of the parties become more concentrated near the mean. On the other side, if the parameters decrease, there is a higher fraction of the population near the center (i.e. 0) and the respective extreme. Note that due to the influence function assumed, parties only fail to obtain that fraction of centrist voters. Consequently, if $\alpha = \beta$ for both parties, and these parameters are greater (resp. smaller) for one of them, it obtains a higher (resp. lower) marginal benefit with its expected second leader. Therefore, the party with the greatest (resp. smallest) parameters would have more (less) leaders if the cost is low enough. The advantage may turn in favor of the party with the smallest parameters if the leaders also convince the centrist followers. The obvious explanation behind this fact is that the expected ideal of the most centrist leader is nearer from 0 in a distribution with smaller parameters.

But what if $\alpha \neq \beta$? Even if we cannot generally compare the marginal benefit obtained with the expected second leader for any two different distributions, we can extract some conclusions. For any level of β (resp. α) examined, the marginal benefit of the expected second leader decreases (resp. increases) with $alpha$ (resp. β). Then, if the two parties have a distribution of potential voters with the same α (resp. β), then the one with the greatest β (α), obtains a higher (resp. lower) marginal benefit from its second expected leader. The effect of β seems to be greater than the one of α, which would be consistent with the conclusions extracted from the case where $\alpha = \beta$.

Hence, if the ideals of citizens in the segment $[-1, 0)$ follow an uniform distribution (i.e. a $Beta(1, 1)$ distribution), while the ones in $[0, 1]$ follow a general $Beta(\alpha, \beta)$, determining which party is having more leaders in equilibrium depends on α and β. If $\alpha > \beta$ (resp. $\alpha < \beta$), then the first leader of party B (resp. A) has a greater marginal benefit and, therefore, for large values of C, party B (resp. A) is the only one holding a leader in equilibrium. That is, the first leader is more productive when she arises from the party that is more biased to the center. On the other side, the vote share of the expected second leader is greater in party A (resp. B). That is, the second expected leader is marginally more productive in the party that is more biased to the extreme, which is an effect that we may also find for a greater number of leaders. Hence, for small values of C we would expect that the most extremist party had more leaders. This result confirms the study made in the discrete case, although the advantage of the most centrist party is limited to the first leader, while this was not so in the discrete case.

5 Conclusion

In this paper, we have constructed a model that endogenously explains how two parties in competition choose their respective optimal number of leaders. It is assumed that parties have separate ideology spectrums that do not overlap. Thus, they do not compete for the same voters. Instead of that, they try to optimally reduce their respective abstention. Separating the population in two groups allows us to examine which are the factors that explain why one of the parties might have more leaders than the other one.

First of all, we have fully characterized the equilibrium for the symmetric case where the two separable populations are uniformly distributed. In this case there is a unique equilibrium, where both parties receive the support of the same number of leaders, a number that is decreasing in the cost-benefit ratio of leadership. This result matches the one by Herrera and Martinelli [10]. From the comparative statics, we have seen the first factor that may explain a different number of leaders between parties. If one of them has a smaller cost-benefit ratio (due to cheaper communication channels or higher expected benefits from wining the elections), it will have more leaders in equilibrium. Similarly, if the leaders of one party are more efficient when influencing their followers or, equivalently, if their party has more potential voters, there could be equilibria where such party would decide to use the support of a greater number of leaders. On the other hand, we have proven that the size of the political spectrum may well be neutral under the influence function assumed. However, if the influence function was decreasing in the ideological distance from leaders to voters, the size of the political spectrum would have a negative effect on the optimal number of leaders.

In the discrete case we have seen how an asymmetric distribution of potential voters can also explain why one party has more leaders than the other one. In this setup, the party which is more biased to the center has an advantage, because its leaders are expected to be, initially, more efficient. This greater efficiency comes from the assumption that leaders convince all those citizens who have a more extremist ideology. We can find equilibria where the most centrist biased party has more leaders, which hold for large values of the leadership cost. However, we can also find some equiibria where the disadvantaged party (the one with a structure biased to the extreme) has more leaders. The latter kinds of equilibria hold for small values of the leadership cost. This effect is very important, because there are cases where the initially disadvantaged party has a greater probability of winning the elections, thanks to its greater number of leaders.

We have checked for the consistency of the conclusions obtained in section 3 with the wide family of beta distribution. With respect to the first leader for each of the parties, the party with the distribution from which the relatively most centrist leaders arise, obtains it for higher leadership costs. The conclusions for a potential second leader are more complex. In case of symmetric populations for each of the two parties ($\alpha = \beta$), the one with the smaller variance obtains a higher marginal benefit and, therefore, will be ready to have more leaders. In case of non-symmetric beta distributions for each of the two parties, the parameter α (resp. β) increases (resp. *decreases*) the marginal benefit for the first leader, while it decreases (resp. increases) the marginal benefit for the second one (supposedly for each of the following ones). Hence, the party facing has a distribution of citizens from which more centrist leaders arise (higher α) has, in equilibrium, its first leader for higher leadership costs. Then, when leadership is very costly this may be the only party with a leader. Whenever leadership costs decrease we would expect the party from which most extremist leaders arise has the greatest number of leaders. This is consistent with the conclusions derived from the discrete case, although it only allows for the party with the relatively centrist leaders having a greater marginal benefit for its first leader. That was not necessarily true in the discrete case, where we saw that the second leader from the most centrist party could have higher marginal benefits.

Both the continuous and the discrete (parties with internal structures) cases lead to a common result: more centrist parties have more leaders at high leadership costs, while the extremist ones look for the support of a greater number of leaders when the leadership costs are low. However, while in the continuous case the most centrist leader only has the greatest number of leaders in an equilibrium $(1,0)$[9], discretization allows more cases. In the case of three groups, we have, for instance, an equilibrium where the centrist party has two leaders, while the opposition only looks for the support of one.

We have found some different factors that may explain why two parties may be supported by a different number of leaders. Among these there are: different leadership costs; different efficiency or size between parties and asymmetric distributions. We have also seen how a different size of parties' political spectrum cannot explain asymmetries in the number of leaders under the influence function assumed. In case the influence was decreasing in the ideological space, the party with the greatest size of political spectrum would be disadvantaged and would have less leaders in equilibrium.

However, there are still some of them that would need to be considered. Given the influence mechanism that we have used, convincing to a potential leader of one party to vote for the opposite one was not possible. In case leaders were able to influence in the two directions of the ideological space there could be found more factors to explain asymmetries in the number of leaders. As it is explained in the expansion, when we assumed such an influence function most of the results in section 2 would still hold. However, the size of the ideological spectrum would, in that case, imply a disadvantage for the party with the largest ideological spectrum. This is explained in the expansion.

[9]if the centrist party is A, otherwise we are referring to $(0,1)$.

For future research, a similar model to this could be applied to real distributions of population. After constructing a distribution of the population over the ideological space, the leaders' marginal benefits could be approximated. This could be interesting in order to examine which party wants to decrease the leadership costs, by, for instance, increasing the benefits from leadership. This could be useful from the normative perspective as well. If smaller leadership costs make extremist parties have more leaders, giving to many facilities to parties' political campaigns could benefit extremist parties. This is a consequence that could be undesirable for society.

A welfare analysis could also be interesting. We could do that from the point of view of citizens, by specifying how is going to be constructed the final policy, how does it change with the different leadership costs, distribution or internal structures, and ordering them with respect to the preferences of the median voter. We could also analyze which are the parties' preferences over some common leadership cost and/or if they are ready to modify their internal structure.

Finally, as an attempt to internalize the cost, the risk of split-up in parties could be considered. Some primaries could be assumed among leaders inside each of the two parties. Once the winning candidate is defined, he could buy the support of the rest of leaders by offering them positions in the future office at a cost in the support of his own fans. Not making an offer good enough to the other leaders would imply a risk of split-up in the party, where the loosing leader would ask to his supporters not to vote for the candidate of their common party. A model with these features could help to identify the factors that may increase parties' risk of split-up.

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FIGURES

Figure 1: Areas of influence of each lider

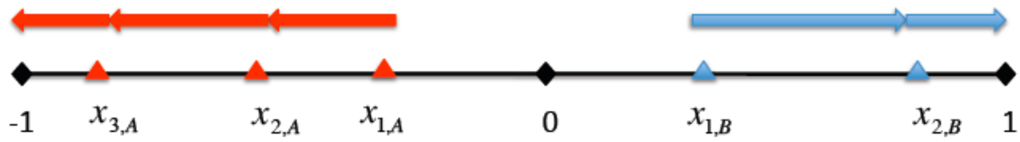


Figure 2: Vote share of the first leader for $Beta(\alpha, \beta)$

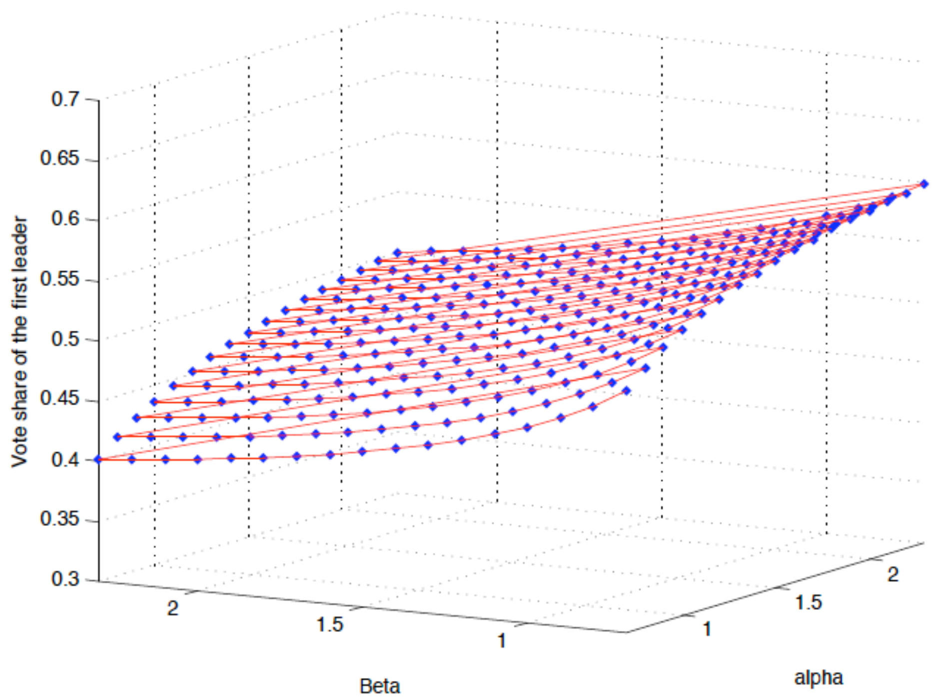
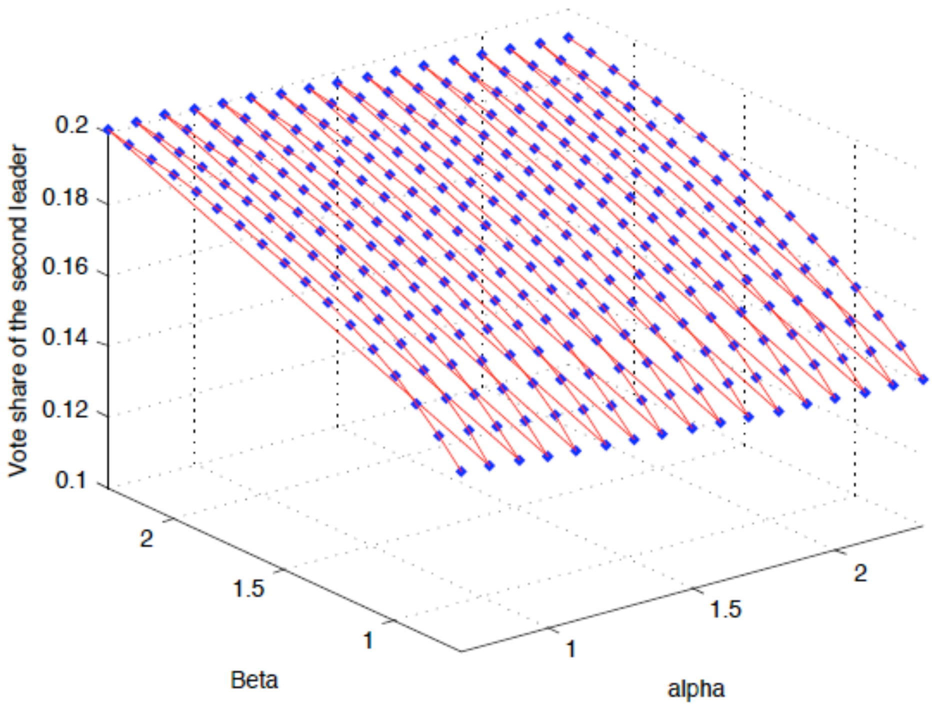


Figure 3: Vote share of the second leader for $Beta(\alpha, \beta)$



APPENDIX

Proof of Lemma 1

Take without loss of generality the leftist party.

$$\frac{MB_A(L_A+1,L_B)}{MB_A(L_A,L_B)} = \frac{L_B/[(L_A+L_B+1)(L_A+L_B)]}{L_B/[(L_A+L_B)(L_A+L_B-1)]}$$
$$= \frac{L_A+L_B-1}{L_A+L_B+1} < 1$$

Proof of Lemma 2

Take without loss of generality the leftist party.

$$\frac{MB_A(L_A,L_B+1)}{MB_A(L_A,L_B)} = \frac{(L_B+1)/[(L_A+L_B+1)(L_A+L_B)]}{L_B/[(L_A+L_B)(L_A+L_B-1)]} = \frac{(L_B+1)(L_A+L_B-1)}{L_B(L_A+L_B+1)}$$

If $\frac{MB_A(L_A,L_B+1)}{MB_A(L_A,L_B)} \geq 1$, it must be the case that:

$$(L_B+1)(L_A+L_B-1) \geq L_B(L_A+L_B+1)$$
$$L_BL_A + L_B^2 - L_B + L_A - L_B - 1 \geq L_AL_B + L_B^2 - L_B$$
$$L_A \geq L_B + 1$$

Proof of Lemma 3

First we can analyze $(1,0)$ (and equivalently $(0,1)$). Party A would never increase its number of leaders, once in $(1,0)$, since it is already winning the elections for sure. As we have seen in the requirement for equilibrium it must be the case that $MB_A(1,0) \geq C$. We have seen that $MB_A(1,0) = \frac{1}{2}$, so it needs to be the case that $C \leq \frac{1}{2}$. However, we should also check for the optimality of party B. At $(1,0)$ party B is losing the elections for sure, while at $(1,1)$ its probability of winning is $\frac{1}{2}$. Then it is easy to conclude that $MB_B(1,1) = \frac{1}{2}$. In order $(1,0)$ to be an equilibrium such move cannot be optimal for party B, so it must be the case that $C > \frac{1}{2}$. This is a contradiction with the previous requirement, so it cannot be the case.

Clearly, no equilibrium of the form $(L,0)$ or $(0,L)$ can exist for $L > 1$, because the party with a positive number of leaders would always be ready to decrease, at least by one, the number of leaders. That would imply a

smaller cost, without diminishing the probability of winning, which is equal to 1. Hence, any situation where there are a strictly positive number of leaders for one party and no leaders for the other one, cannot constitute an equilibrium.
The first part of the proof has already given before. That is, no equilibrium of the form $(L, 0)$ or $(0, L)$ exists for any $L \geq 0$. Now, we will prove that no equilibrium exists when both parties have a positive number of leaders and one has more leaders than the other. In order to do that, take without loss of generality a pair of leaders $(L+j, L)$, where j is a strictly positive integer and check for the equilibrium conditions.

In order to be optimal for party A, it must be the case that $MB_A(L+j, L) \geq C$ and $MB_A(L+j+1, L) < C$. That means:

$$\frac{L}{(2L+j)(2L+j-1)} \geq C \text{ and } \frac{L}{(2L+j+1)(2L+j)} < C$$

And in order to be optimal for party B, we would equivalently ask that $MB_B(L+j, L) \geq C$ and $MB_B(L+j, L+1) < C$, which is resumed in:

$$\frac{L+j}{(2L+j)(2L+j-1)} \geq C \text{ and } \frac{L+j}{(2L+j+1)(2L+j)} < C$$

However, $MB_B(L+j, L+1) \geq MB_A(L+j, L)$, as it is proved here:

$$\frac{L+j}{(2L+j+1)(2L+j)} - \frac{L}{(2L+j)(2L+j-1)} \geq 0$$

$$\frac{(L+j)(2L+j-1) - L(2L+j+1)}{(2L+j+1)(2L+j-1)} \geq 0$$

Since the two terms in the denominator are always positive the inequality holds as long as the numerator is positive. So we ask that:

$$(L+j)(2L+j-1) - L(2L+j+1) = 2L^2 + jL - L + 2Lj + j^2 - j - 2L^2 - jL - L \geq 0$$

$$2Lj + j^2 - j - 2L \geq 0$$

$$2L(j-1) + j(j-1) \geq 0$$

which holds for $j \geq 1$, so $MB_B(L+j+1, L) \geq MB_A(L+j, L) \geq C$. Then it cannot be true that $MB_B(L+j+1, L) < C$.

Proof of Proposition 2

In order to keep the equal size of the parties we must redefine the density function of $x_{1,B}$ and the joint density function of $(z_{1,A}, x_{1,B})$, which are respectively:

$$f_{x_{1,B}}(x_{1,B}) = \frac{L_B}{\alpha}\left(1 - \frac{x_{1,B}}{\alpha}\right)^{L_B-1}$$

$$f_{z_{1,A},x_{1,B}}(z_{1,A}, x_{1,B}) = \frac{L_A L_B z_A^{L_A-1}}{\alpha}\left(1 - \frac{x_B}{\alpha}\right)^{L_B-1}$$

The first thing we will do is calculating the probability of winning of party A. This is given by the probability of $z_{1,A}$ being greater than $\frac{\alpha - x_{1,B}}{\alpha}$, which is given by the following integral:

$$P_A(L_A, L_B) = \int_0^\alpha \int_{\frac{\alpha - x_{1,B}}{\alpha}}^1 \frac{L_A L_B z_{1,A}^{L_A-1}}{\alpha}\left(1 - \frac{x_{1,B}}{\alpha}\right)^{L_B-1} dz_{1,A}dx_{1,B}$$

Through an easy computation of the preceding integral:

$$\int_0^\alpha \int_{1-\frac{x_{1,B}}{\alpha}}^1 \frac{L_A L_B z_{1,A}^{L_A-1}}{\alpha}(1 - \frac{x_{1,B}}{\alpha})^{L_B-1} dz_{1,A}dx_{1,B} =$$

$$\int_0^\alpha (1 - \frac{x_{1,B}}{\alpha})^{L_B-1}\frac{L_B}{\alpha}[1 - (1 - \frac{x_{1,B}}{\alpha})^{L_A}]dx_{1,B} =$$

$$\int_0^\alpha (1 - \frac{x_{1,B}}{\alpha})^{L_B-1}\frac{L_B}{\alpha}dx_{1,B} - \int_0^\alpha (1 - \frac{x_{1,B}}{\alpha})^{L_A+L_B-1}\frac{L_B}{\alpha}dx_{1,B} =$$

$$-\frac{(1-\frac{0}{\alpha})^{L_B}}{L_B\frac{-1}{\alpha}}\frac{L_B}{\alpha} + \frac{(1-\frac{0}{\alpha})^{L_A+L_B}}{(L_A+L_B)\frac{-1}{\alpha}}\frac{L_B}{\alpha} =$$

$$1 - \frac{L_B}{L_A+L_B} = \frac{L_A}{L_A+L_B}$$

we obtain that $P_A(L_A, L_B) = \frac{L_A}{L_A+L_B}$. Since the probability of winning for party A is exactly the same as in the previous section, lemmas 1, 2, 3 and proposition 1 hold as exactly as before. Therefore:

Proof of Lemma 4

Assume, without loss of generality that $C_A > C_B$ and that (L_A, L_B) is an equilibrium. Then, the marginal benefit of party A has to be greater than C_A, that is: $MB_A(L_A, L_B) = \frac{L_B}{(L_A+L_B)(L_A+L_B-1)} \geq C_A$. At the same time, the marginal benefit of party B has to be greater than C_B; but, not only that, the marginal benefit, having one more leader has to be strictly smaller than C_B, that is: $Q_B(L_A, L_B+1) = \frac{L_A}{(L_A+L_B+1)(L_A+L_B)} < C_B$.

If $C_A > C_B$ it has to be the case that $MB_A(L_A, L_B) > MB_B(L_A, L_B + 1)$. That is:

$$\frac{L_B}{(L_A+L_B)(L_A+L_B-1)} > \frac{L_A}{(L_A+L_B)(L_A+L_B+1)}$$

The preceding inequality only holds if $L_B^2 - L_B - L_A^2 + L_A > 0$, or, equivalently, if $L_B^2 - L_A^2 > L_B - L_A$. This condition can also be expressed this way:

$$(L_B + L_A)(L_B - L_A) > L_B - L_A$$

This inequality cannot hold for $L_A > L_B$. Both sides of the inequality would be negative and the LHS would be obviously greater in absolute value. Therefore, in equilibrium it needs to be the case that $L_B \geq L_A$.

Proof of Proposition 3

Assume that the A party has L leaders and the B party has $L+j$ leaders. If that is an equilibrium it must be the case that $MB_B(L, L+j) \geq C$ and, among others, that $MB_A(L+1, L+j) < C$. We will compute:

$$MB_A(L+1, L+j) - MB_B(L, L+j) = \frac{(L+j)[1+(\beta-1)(L+1)+(\beta-1)(L+j)]}{(2L+j+1)\beta} - \frac{L}{2L+j-1}$$

$$= \frac{(L+j)(2L+j-1)[1+(\beta-1)(L+1)+(\beta-1)(L+j)] - L(2L+j+1)\beta}{(2L+j+1)(2L+j-1)\beta}$$

Which is weakly greater than 0 as long as the numerator is so. Then we ask that:

$$(L+j)(2L+j-1)[1+(\beta-1)(L+1)+(\beta-1)(L+j)] - L(2L+j+1)\beta \geq 0$$

$$(2L^2+jL-L+2Lj+j^2-j)[\beta+2L(\beta-1)+j(\beta-1)] - \beta[2L^2+(j+1L)] \geq 0$$

$$[2L^2+L(3j-1)+j(j-1)][\beta+2L(\beta-1)+j(\beta-1)] - \beta[2L^2+(j+1L)] \geq 0$$

Which is true because for $j \geq 1$ and $j(j-1) \geq 0$.

Calculus of $E[|x_{i+1,j} - x_{i,j}|]$, where $x_{i,j}, x_{i+1,j} \sim U(0, \alpha)$

$$E[|x_{i+1,j} - x_{i,j}|] = \int_0^\alpha \int_0^{x_{i+1,j}} (x_{i+1,j} - x_{i,j}) \frac{L_j!(\frac{x_{i,j}}{\alpha})^{i-1}(1-\frac{x_{i+1,j}}{\alpha})^{L_j-i-1}}{\alpha^2(i-1)!(L_j-i-1)!}$$

$$= \int_0^{\alpha}\int_0^{x_{i+1,j}} \frac{L_j! x_{i+1,j}^{i+1}(\alpha - x_{i+1,j})^{L_j-i-1}}{\alpha^{L_j} i!(L_j-i-1)!} - \int_0^{\alpha}\int_0^{x_{i+1,j}} \frac{L_j! x_{i+1,j}^{i+1} i(\alpha - x_{i+1,j})^{L_j-i-1}}{\alpha^{L_j}(i+1)!(L_j-i-1)!}$$

$$= \int_0^{\alpha} \frac{x_{i+1,j}^{L_j}(i+1)}{\alpha^{L_j}} - \int_0^{\alpha} \frac{i x_{i+1,j}}{\alpha^{L_j}} = \frac{\alpha(i+1)}{L_j-1} - \frac{\alpha i}{L_j-1} = \frac{\alpha}{n+1}$$

Same number of leaders when B is very biased to the center.

$(1,1)$ can be an equilibrium, if $MB_A(1,1) = 1 - \frac{5r}{6} \geq C$, $MB_B(1,1) = \frac{5r}{6} \geq C$, $MB_A(2,1) = \frac{r}{6} < C$ and $MB_B(1,2) = (1-r)\frac{5}{6} - \frac{st}{2(r+t)} - \frac{st}{2(r+s)} < C$. We can easily verify this assertion by assuming for instance $t = 0$ and $s = 0.3$. Since, $\min\{MB_A(1,1), MB_B(1,1)\} = \frac{5}{12} > \max\{MB_A(2,1), MB_B(1,2\} = \frac{1}{4}$, there are values of C for which the equilibrium exists.

$(3,3)$ can be an equilibrium as well. This is direct by considering $t > 0$ and a leadership cost (C) sufficiently small.

It is also argued that $MB_B(1,2) > MB_B(3,3)$. This is because: $(1-r)\frac{5}{6} > \frac{s+t}{2} > \frac{st}{r+t} + \frac{st}{r+s}$.

More leaders for party A when B is very biased to the center

$(3,2)$ can be an equilibrium. For that, it must be the case that $MB_A(3,2) = \frac{1}{6} \geq C$, $MB_B(3,2) = (1-r)\frac{1}{2} - \frac{st}{2(r+t)} - \frac{st}{2(r+s)} \geq C$ and $MB_B(3,3) = \frac{st}{2(r+t)} + \frac{st}{2(r+s)}$¡C. This can be true, for instance, for $\{r,s,t\} = \left\{\frac{7}{10}, \frac{2}{10}, \frac{1}{10}\right\}$, where $MB_B(3,2) = \frac{91}{720}$ and $MB_B(3,3) = \frac{17}{720}$.

If any of the two parties has incentives to deviate the equilibria cannot hold. That is why $(3,1)$ and $(3,2)$ cannot coexist. It is straightforward that $(3,2)$ is an equilibrium for strictly smaller values of C than $(3,1)$.

More leaders for party B when B is very biased to the center.

$(0,1)$ can be an equilibrium. In order to prove such existence it is sufficient to find some values of $\{r,s,t\}$ for which $MB_B(0,1) = \frac{1}{2} > MB_A(1,1) = 1 - \frac{5r}{6}$. This is obviously the case for values of r sufficiently close to 1.

$(1,2)$ can be an equilibrium. In order to be so, we must have $MB_A(1,2) = \frac{1}{6} + \frac{st}{2(r+t)} + \frac{st}{2(r+s)} \geq C$, $MB_B(1,2) = (1-r)\frac{5}{6} - \frac{st}{2(r+t)} - \frac{st}{2(r+s)}$, $MB_A(2,2) = \frac{1}{6} < C$ and $MB_B(1,3) = \frac{st}{2(r+t)} + \frac{st}{2(r+s)} < C$. Take, for instance $\{r,s,t\} = \left\{\frac{7}{10}, \frac{2}{10}, \frac{1}{10}\right\}$. Since $\min\{MB_A(1,2), MB_B(1,2)\} > \max\{MB_A(2,2), MB_B(1,3)\}$, there are values of C for which $(1, 2)$ holds as equilibrium.

$(1, 3)$ and $(2, 3)$ are never an equilibrium because $MB_A(1, 3) = MB_A(2, 3) = MB_A(3, 3) = \frac{1}{6}$. Then if it we are in the case that $MB_A(1, 3) \geq C$, then it is also true that $MB_A(2, 3) = MB_A(3, 3) \geq C$ and party A should have three leaders.

$(0, 1)$ and $(1, 2)$ cannot hold as equilibrium at the same time, because if $(0, 1)$ is an equilibrium, then $MB_A(1, 1) < C$. However, $MB_A(1, 1) > MB_A(1, 2)$ (which also implies that $(1, 2)$ is an equilibrium for smaller values of C). This is because:

$$MB_A(1, 1) = 1 - \frac{5r}{6} > \frac{1}{6} - \frac{st}{2(r + t)} - \frac{st}{2(r + s)} = MB_A(1, 2)$$

$$\frac{5}{6} > \frac{5}{6}r - \frac{st}{2(r + t)} - \frac{st}{2(r + s)}$$

Proof of lemma 5. Uniqueness of the equilibrium when B is very biased to the center.

We have already stated that whenever $(0, 0)$ is an equilibrium (i.e. when $C > \frac{1}{2}$) it is unique. If a pair of leaders is an equilibrium, the deviation cannot be optimal and so, two pairs of leaders that differ one from another in one leader for one of the parties cannot be equilibrium at the same time. This is the case of : $(1, 1)$ and $(0, 1)$; $(1, 1)$ and $(1, 2)$; $(3, 3)$ and $(3, 2)$

$(1, 1)$ and $(3, 1)$ cannot be equilibria at the same time. In case they were, two of the conditions that would need to be fulfilled are $MB_A(2, 1) < C$ and $MB_A(3, 1) \geq C$. However, this is not possible, since $MB_A(2, 1) = MB_A(3, 1)$. A similar argument also applies to $(1, 1)$ and $(3, 2)$. If these two were equilibria at the same time, that would require $MB_B(1, 2) < C$ and $MB_B(3, 2) \geq$, which cannot be true because $MB_B(1, 2) > MB_B(3, 2)$.

$(3, 3)$ and $(3, 1)$ cannot be equilibria at the same time either. If they were $MB_B(3, 3) \geq C$ and $MB(3, 2) < C$. However, we have already checked that $MB_B(3, 2) > MB_B(3, 3)$. $(3, 3)$ and $(0, 1)$ cannot coexist as equilibria because if they did, $MB_A(1, 1) < C$ and $MB_B(3, 3)$ would need to be true at the same time. However:

$$MB_A(1, 1) = 1 - \frac{5r}{6} > s + t > \frac{st}{2(r + t)} + \frac{st}{2(r + s)} = MB_B(3, 3)$$

$(1, 2)$ and $(0, 1)$ cannot coexist as equilibria. This is because $MB_A(1, 1) > MB_B(1, 2)$, which is straightforward. $(1, 2)$ cannot coexist with $(3, 1)$, because $MB_A(2, 2) = \frac{1}{6} > \frac{r}{6} = MB_A(3, 1)$. A similar argument is used to justify that $(1,2)$ and $(3,2)$ cannot be equilibria at the same time, that is, $MB_A(2,2) = \frac{1}{6} = MB_A(3,2)$.

$(3,1)$ and $(0,1)$ cannot coexist because $MB_A(1,1) > MB_A(3,1)$. Similarly, $(3,1)$ and $(1,2)$ cannot be equilibrium at the same time, since $MB_A(2,2) > MB_A(3,1)$.

Again, $(3,2)$ cannot be equilibria at the same time that $(0,1)$ or $(1,2)$, Because $MB_A(1,1) > MB_A(3,2)$ and $MB_A(2,2) > MB(3,2)$.

Same number of leaders when B is not so much biased to the center.

$(1,1)$ can be an equilibrium if $MB_A(1,1) = \frac{r}{6} + \frac{2s}{3} + t \geq C$, $MB_B(1,1) = \frac{5r}{6} + s3 \geq C$, $MB_A(2,1) = \frac{r}{6} + \frac{s}{3} < C$ and $MB_B(1,2) = \frac{s}{2} + \frac{5t}{6} - \frac{st}{3(r+t)} - \frac{st}{3(r+s)} < C$. We can easily verify this assertion by assuming $\{r,s,t\} = \{\frac{6}{10}, \frac{3}{10}, \frac{1}{10}\}$. Since, $\min\{MB_A(1,1), MB_B(1,1)\} = \frac{4}{10} > \max\{MB_A(2,1), MB_B(1,2\} = \frac{131}{630}$, there are values of C for which the equilibrium exists.

$(2,2)$ can also be an equilibrium if $MB_A(2,2) = \frac{1}{6} + \frac{st}{6(r+t)} + \frac{st}{6(r+s)} \geq C$, $MB_B(2,2) = \frac{2s}{3} + \frac{2t}{3} - \frac{st}{2(r+t)} - \frac{st}{2(r+s)} \geq C$, $MB_A(3,2) = \frac{1}{6} < C$ and $MB_B(2,3) = \frac{st}{2(r+t)} + \frac{st}{2(r+s)} < C$. Assume, for instance, $\{r,s,t\} = \{\frac{6}{10}, \frac{3}{10}, \frac{1}{10}\}$. Since, $\min\{MB_A(2,2), MB_B(2,2)\} = \frac{121}{630} > \max\{MB_A(3,2), MB_B(2,3\} = \frac{8}{315}$, there are values of C for which the equilibrium exists. $(2,2)$ and $(1,1)$ can never be equilibria at the same time, because $MB_B(2,2) > MB_A(2,1)$. That is:

$$\frac{s}{3} + \frac{t}{3} + \frac{t}{3}\left[1 - \frac{st}{r+t} - \frac{st}{r+s}\right] > \frac{r}{6}$$

On the one side if $s + t > \frac{1}{3}$, then $\frac{s}{3} + \frac{t}{3} + \frac{t}{3} > \frac{r}{6}$ and on the other, $\frac{st}{r+t} + \frac{st}{r+s} < 1$.

$(3,3)$ can obviously be an equilibrium as long as $t > 0$, for values of C small enough. $(3,3)$ and $(1,1)$ or $(2,2)$ cannot be equilibria at the same time. This is obvious because $MB_A(3,3) = \frac{1}{6} = MB_A(3,2)$, while $MB_A(2,1) > \frac{1}{6}$.

More leaders for party A when B is not much biased to the center.

$(1,0)$ can be an equilibrium. In order to be so, take for instance $\{r,s,t\} = \{\frac{4}{10}, \frac{31}{100}, \frac{29}{100}\}$, then $MB_B(1,1) < \frac{1}{2}$.

$(3,1)$ and $(3,2)$ can be equilibria as long as we choose values of s and t conveniently close to 0.

$(2,1)$ cannot be an equilibrium, because $MB_B(2,2) > MB_A(2,1)$, as we have proven before.

$(3,1)$ and $(3,2)$ cannot coexist as equilibria by construction. On the other side, $(1,0)$ cannot coexist with $(3,1)$ nor with $(3,2)$, because clearly $MB_B(1,0) > MB_B(3,2)$ and $MB_B(1,0) > MB_B(3,3)$.

More leaders for party B when B is not so much biased to the center.

$(0,1)$ can be an equilibrium. Take $\{r,s,t\} = \left\{\frac{6}{10}, \frac{31}{100}, \frac{29}{100}\right\}$

$(1,2)$ can be also be an equilibrium. Take for instance $\{r,s,t\} = \left\{\frac{2}{5}, \frac{31}{100}, \frac{29}{100}\right\}$. Then, $MB_B(1,2) > MB_A(2,2)$ and the rest of necessary inequalities hold straightforward.

$(1,3)$ and $(2,3)$ cannot be equilibrium because in any of them the marginal probability of party A is $\frac{1}{6}$. Since $MB_A(3,3) = \frac{1}{6}$, if being at $(1,3)$ or at $(2,3)$ is good for party A, it should deviate to $(3,3)$.

$(0,1)$ and $(1,2)$ cannot obviously coexist as equilibria, because $MB_A(1,1) > MB_A(1,2)$.

Proof of Lemma 6. Uniqueness of the equilibrium when party B is a bit biased to the center.

We have already compare those equilibria belonging to the same group (equal number of leaders, more for party A or more for party B) obtaining that they could not coexist with each other. By definition of the equilibrium the following pairs cannot be so at the same time: $(1,1)$ and $(1,2)$; $(2,2)$ and $(1,2)$; $(2,2)$ and $(3,2)$; $(3,3)$ and $(3,2)$.

$(1,1)$ cannot coexist with $(3,1)$ or $(3,2)$, because $MB_A(2,1) > \frac{1}{6}$. Similarly, $(2,2)$ cannot coexist with $(3,1)$, since $MB(3,2) = \frac{1}{6}$.

Moreover, $(3,3)$ and $(1,2)$ cannot coexist because $MB_A(2,2) > MB_B(3,3)$. That is, $\frac{1}{6} + \frac{st}{6(r+t)} + \frac{st}{6(r+s)} > \frac{st}{2(r+t)} + \frac{st}{2(r+s)}$, because:

$$\frac{1}{6} > \frac{2s}{6} = \frac{2s(r+t)}{6(r+t)} > \frac{4st}{6(r+t)} > \frac{2st}{6(r+t)} + \frac{2st}{6(r+s)}$$

$(3,3)$ and $(3,1)$ cannot be equilibria at the same time, because $MB_(3,2) > MB_(3,3)$. This is straightforward, as we note that $s+t > \frac{2st}{r+t}$.

Finally $(1,2)$ cannot coexist with $(3,1)$ or $(3,2)$, because $MB_A(2,2) \geq \frac{1}{6}$.

Possible equilibria where B is (not so much) biased to the extreme.

$(1,1)$ can be an equilibrium. Consider $\{r,s,t\} = \left\{\frac{8}{25}, \frac{33}{100}, \frac{7}{20}\right\}$. Then, party B, which enjoys the smallest marginal benefit is obtaining $MB_B(1,1) =$

$\frac{181}{300}$ is greater than the marginal benefit of increasing one leader for any of the two parties. Then, there must exist one $C > 0$ for which $(1,1)$ is an equilibrium.

$(3,3)$ can be obviously be an equilibrium for $r > 0$ as long as we choose a sufficiently small value of C.

$(1,0)$ can be an equilibrium. For this it must be the case that $MB_B(1,1) < \frac{1}{2}$, which is true because $MB_A(1,1) > \frac{1}{2}$. $(3,2)$ can be an equilibrium as well. In order to check for that, consider $\{r,s,t\} = \left\{\frac{8}{25}, \frac{33}{100}, \frac{7}{20}\right\}$. $MB_B(3,2) > MB_B(3,3)$ is clear. Furthermore, $MB_A(3,2) = \frac{122}{585} > \frac{49}{390} = MB_B(3,3)$.

. Proof of Lemma 7. Uniqueness of the equilibrium when party B is a bit biased to the center

By definition of equilibrium, $(1,0)$ and $(1,1)$, $(0,1)$ and $(1,3)$ or $(3,2)$ and $(3,3)$ cannot coexist.

$(1,1)$ and $(3,3)$ cannot be equilibria at the same time because $MB_B(1,2) > MB(3,3)$.

$(1,0)$ cannot coexist with $(3,2)$ or $(3,3)$, because $MB_B(1,1) > MB_A(3,2) > MB_A(3,3)$. Similarly, $(0,1)$ cannot coexist with $(3,2)$ or $(3,3)$, since$MB_A(1,1) > MB_A(3,2) > MB_A(3,3)$.

EXTENSION
Influencing in the two directions

Up to now, we have been assuming that leaders could only influence to those citizens that were more extreme than them. In this extension, however, we will consider that leaders can influence in both directions of the ideological space. This would be equivalent to assume that citizens had single-peaked preferences and vote for the party that is supported by the leader they like the most. In this case those citizens with centrist ideal policies also vote.

The first calculus, which we used to prove the existence and the uniqueness of equilibrium, would be identical. Remember that the objective of one party is maximizing its winning probability. Take without loss of generality party A. Under symmetric single peaked preferences, parties would obtain the vote of all citizens with an ideal policy nearer from their most centrist leader. Therefore, party A's winning probability is equal to $P(x_{1,A}+x_{1,B} > 0)$, which is equivalent to $P(z_{1,A} > 1-x_{1,B})$, which is the one we calculated, previously. Therefore, we would expect the same unique equilibrium, even with single-peaked preferences.

Different size

When we analyzed how parties maximized the votes they could ensure, this could have a double interpretation. Either we could assume that one of the parties has a larger potential electorate or that leaders of one of the parties are more efficient. These two interpretations, although treated differently, would give us qualitatively similar results.

If we assumed that leaders from party A were more effective while those of party B only obtained a fraction $\frac{1}{\beta}$ of the votes (where $\beta > 1$). Then the winning probability for party A would be equal to:

$$P\left(\frac{x_{1,A}+x_{1,B}}{2} > \left[1-\frac{x_{1,A}+x_{1,B}}{2}\right]\frac{1}{\beta}\right)$$

which is equivalent to:

$$P\left(z_{1,A} > (1-x_{1,A})\frac{2}{2+\beta}\right)$$

This would yield a qualitatively equivalent result, because we can rename the inefficiency by $\frac{1}{\gamma} = \frac{2}{2+\beta}$. It is straightforward to prove that the inefficiency is larger in this case whenever $2 > \beta$.

If, on the other side, we assumed that the number of people with an ideal policy greater than 0 is strictly smaller than those with a negative ideal, the approach is slightly different. Assume that those with an ideal policy in $[0,1]$ have a total size of $\frac{1}{\beta}$ (where $\beta > 1$), while those with an ideal policy in $[-1,0)$ have a size equal to 1.

If $z_{1,A} > 1 - x_{1,B}$ (which occurs with probability $\frac{L_A}{L_A+L_B}$), then $P_A(L_A, L_B) = 1$. On the other side, if $z_{1,A} \leq 1 - x_{1,B}$, the winning probability for party A is equal to:

$$P\left(1 - \frac{x_{1,A} + x_{1,B}}{2} > \frac{1}{\beta} + \frac{x_{1,A} + x_{1,B}}{2}\right)$$

which is equivalent to $P\left(x_{1,B} + z_{1,A} < 2 - \frac{1}{\beta}\right)$. In order to calculate this, we can just obtain:

$$P(x_{1,B} + z_{1,A} < t) = \int_0^1 \int_0^{t - z_{1,A}} L_A L_B (1 - x_{1,B})^{L_B - 1} z_{1,A}^{L_A - 1} dx_{1,B} dz_{1,A} =$$

$$= 1 - \int_0^1 L_A z_{1,A}^{L_A - 1} (z_{1,A} + 1 - t)^{L_B} dz_{1,A} = 1 - \sum_{i=0}^{L_B} \frac{L_B! L_A! (1-t)^{L_B + i}}{(L_A + i)!(L_B - i)!}$$

Therefore,

$$P\left(x_{1,B} + z_{1,A} < 2 - \frac{1}{\beta}\right) = 1 - \sum_{i=0}^{L_B} \frac{L_B! L_A! \left(\frac{1}{\beta} - 1\right)^{L_B + i}}{(L_A + i)!(L_B - i)!}$$

And the party A's winning probability:

$$P_A(L_A, L_B) = \frac{LA}{L_A + L_B} + \frac{L_B}{L_A + L_B}\left[1 - \sum_{i=0}^{L_B} \frac{L_B! L_A! \left(\frac{1}{\beta} - 1\right)^{L_B + i}}{(L_A + i)!(L_B - i)!}\right]$$

Therefore, party A's marginal benefit equals: $MB_A(L_A, L_B) = \frac{L_B}{(L_A+L_B)(L_A+L_B-1)} + \sum_{i=0}^{L_B} \frac{L_B! \left(\frac{1}{\beta}-1\right)^{L_B - i}}{L_B + i} \left[\frac{(L_A-1)!}{(L_A+i-1)!(L_A+L_B-1)} - \frac{L_A!}{(L_A+i)!(L_A+L_B)}\right]$

Clearly, the term inside the brackets is always positive, so the marginal benefit for party A is strictly greater compared to the one it obtained for the same number of leaders under complete symmetry. The same calculus applied to party B yields a negative term, confirming that party A is going to obtain a higher marginal benefit than B and, therefore, a weakly greater number of leaders in equilibrium.

Different size of the political spectrum

We will consider now that the potential leaders from party B arise from the uniform $[0, \alpha]$, while the number of citizens with an ideal in $[-1, 0)$ is exactly the same than those who have it in $[0, 1]$. Then, party A's winning probability $(P_A(L_A, L_B))$ is equal to:

$$\int_0^\alpha \int_{1-x_{1,B}}^1 L_A L_B z_{1,A}^{L_A-1} \left(1 - \frac{x_{1,B}}{\alpha}\right)^{L_B-1} \frac{1}{\alpha} dz_{1,A} dx_{1,B}$$

$$= 1 - \int_0^\alpha \frac{1}{L_B} \left(1 - \frac{x_{1,B}}{\alpha}\right)^{L_B-1} (1 - x_{1,B})^{L_A} dx_{1,B}$$

$$= 1 - 1 - \int_0^\alpha - \int_0^\alpha L_A (1 - x_{1,B})^{L_A-1} (1 - x_{1,B})^{L_B} dx_{1,B}$$

$$= \sum_{i=1}^{L_A} \frac{\alpha^i L_A! L_B!}{(L_A - i)!(L_B + i)!}$$

Hence, the marginal benefit for party A:

$$MB_A(L_A, L_B) = \frac{\alpha^{L_A} L_A! L_B!}{(L_A + L_B)!}$$

This marginal benefit is increasing in α, so we would have a different result with respect to the one obtained for the case where parties have no believes about citizens' preferences and they only take into account the votes they can ensure with respect to the ideals' distribution. The party with the largest political spectrum will suffer from disadvantage in obtaining the votes from the centrist citizens.