GAME THEORY
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Problem Set 2
(to be returned on Thursday, September 21th)

EXERCISE 1. (1.3 F&T (a)) (*only part (a)!*) (Nash demand game)

EXERCISE 2. (1.7 F&T) (public good)

EXERCISE 3. (1.12 F&T)

EXERCISE 4. (8.C.4 M-C&W&G)

EXERCISE 5. Find the set of rationalizable actions of each player in the following two-player game:

	$b1$	$b2$	$b3$	$b4$
$a1$	0 , 7	2 , 5	7 , 0	0 , 1
$a2$	5 , 2	3 , 3	5 , 2	0 , 1
$a3$	7 , 0	2 , 5	0 , 7	0 , 1
$a4$	0 , 0	0 , -2	0 , 0	10 , -1

EXERCISE 6. Consider a variant of EXERCISE 6 in Problem set 1 in which contributions are restricted to be nonnegative. Show the following:

(a) Any contribution of more than $w_i/2$ is strictly dominated for player i.

(b) If $n = 3$ and $w_1 = w_2 = w_3$ then every contribution of at most $w/2$ is rationalizable.

(c) If $n = 3$ and $w_1 = w_2 < (1/3)w_3$ then the unique strategy of player 3 that survives iterated deletion of strictly dominated strategies is the one in which his contribution is $w_3/2$, and players 1 and 2 contribute 0.

EXERCISE 7. Let us say that a complete information game in strategic form $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ is *perfectly competitive* if $N = \{1, 2\}$, and for every $s, s' \in S$, $u_1(s) \geq u_1(s')$ if and only if $u_1(s') \geq u_1(s)$. Prove that if Γ is perfectly competitive, then:

(a) For every mixed strategies σ and σ', $u_1(\sigma) \geq u_1(\sigma')$ if and only if $u_1(\sigma') \geq u_1(\sigma)$.

(b) If σ and σ' are both Nash equilibria, then $u_i(\sigma) = u_i(\sigma')$ for $i = 1, 2$.

(c) If σ and σ' are both Nash equilibria, then (σ_1, σ'_2) and (σ'_1, σ_2) are also NE.