GAME THEORY
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Problem Set 4
(to be returned on Thursday, September 28th)

EXERCISE 1. Suppose that three players share a pie by using the following procedure. First player 1 proposes a division, then players 2 and 3 simultaneously respond either "yes" or "no". If players 2 and 3 both say "yes" then the division is implemented; otherwise no player receives anything. Each player prefers more of the pie to less. Formulate this situation as an extensive game with simultaneous moves and find its subgame perfect equilibria.

EXERCISE 2. (3.8 F&T)

EXERCISE 3. In the Rubinstein-Stahl Bargaining Model, there are some Nash equilibria in which an agreement is not reached immediately. Can you propose such a NE?

EXERCISE 4. (4.8 (a) F&T)

EXERCISE 5. (A war of attrition game) Two small grocery stores on the same block are feeling the effects of a large supermarket that was recently constructed a half-mile away. As long as both remain in business, each will loss $1000 per month. On the first day of every month, when the monthly rent for the stores is due, each grocer who is still in business must independently decide whether to stay in business for another month or to quit. If one grocer quits, then the grocer who remains will make $500 per month profit thereafter. Assume that, once a grocer quits, his or her lease will be taken by some other merchant (not a grocer), so he or she will not be able to reopen a grocery store in the block, even if the other grocer also quits. Each grocer wants to maximize the expected discounted average value of his or her monthly profits, using a discount factor per month of $\delta = .99$.

(a) Find an equilibrium of this situation in which both grocers randomize between staying and quitting every month until at least one grocer quits.

(b) Suppose now that grocer 1 has a slightly larger store than grocer 2. As long as both stores remain in business, grocer 1 loses \$1200 per month and grocer 2 loses \$900 per month. If grocer 1 had the only grocery store on the block, he would earn \$700 profit per month. If grocer 2 had the only grocery store on the block, she would earn \$400 per month. Find an equilibrium of this situation in which both grocers randomize between staying and quitting every month, until somebody actually quits. In this equilibrium, which grocer is more likely to quit first?

EXERCISE 6. Consider an infinitely repeated game in which the players' preferences are represented by the discounting criterion, the common discount factor is 1/2, and the constituent game is the game:

	A	D
A	2, 3	1, 5
D	0, 1	0, 1

Show that $((A, A), (A, A), \ldots)$ is not a subgame perfect equilibrium outcome path.

EXERCISE 7. (5.1 F&T)

EXERCISE 8. (5.2 F&T)

EXERCISE 9. Consider the three-player symmetric infinitely repeated game in which each player's preferences are represented by the discounting criterion and the constituent game is $\{ \{1, 2, 3\}, (A_i), (u_i) \}$ where for $i = 1, 2, 3$ we have $A_i = [0, 1]$ and

$$u_i(a_1, a_2, a_3) = a_1a_2a_3 + (1–a_1)(1–a_2)(1–a_3) \quad \text{for all } (a_1, a_2, a_3) \in A_1 \times A_2 \times A_3.$$

(c) Find the set of individually rational (or enforceable) payoffs of the constituent game.

(d) Show that for any discount factor $\delta \in (0, 1)$ the payoff of any player in any subgame perfect equilibrium of the repeated game is at least 1/4.

(e) Reconcile these results with the subgame perfect folk theorem for the discounting criterion.