Standing in Line: Demand for investment opportunities with exogenous priorities*

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Abstract

We look at a model in which agents can invest in a project with a limited number of available slots. Agents have incomplete information about the projects expected payoffs. Based on that, they must decide whether to invest in the risky project or take a safe outside option. Slots are assigned following an exogenous priority order. Low priority agents may face a winner's curse: if they choose to invest and obtain a slot in the project it must be that agents with higher priority choose not to do so. In equilibrium, only high priority agents choose to invest when their private information indicates they should. Low priority agents take the outside option independently of their private information. This feature of equilibrium is maintained when we look at variations of the model with priorities assigned by lottery or determined by a Bernoulli process. We perform relevant comparative statics and compare equilibrium outcomes of our simultaneous action model with the ones from a social learning model. Our analysis highlights unexplored links between market design features and the performance of such markets. In particular, agents' knowledge of the priority order affects both demand and efficiency. Furthermore, herding behavior occurs even in the absence of social learning.

Keywords: *market design, incomplete information, limited supply, crowd-funding, assignment mechanism, winner's curse.*

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1 Introduction

Markets for goods or investment opportunities are often characterized by limited supply: opportunities for micro-investments, initial private offerings (IPO's), offers in the housing market, and job offers in the labor market are some examples. When this is the case, interdependencies are created among market participants' actions and outcomes: some agents can obtain access to these opportunities only if others choose not to. If furthermore actions are motivated by the available information, interesting strategic effects are observed. The following "down-to-earth" example should make the nature of these effects clear to the reader.

You come back home after work in the evening and notice an add in the morning's paper offering 10 "Clean-your-house Robots" at a very low price to the first 10 persons to send a free sms to a specific number. At first glance this offer seems appealing. But before sending the sms you think again: given that the add was in the morning's paper and many hours have passed since it was published, the only chance of winning a robot for a low price is if less than 10 persons have sent the sms already. This would happen only if, unlike yourself, the vast majority of readers that saw the add during the day thought that this robot is probably useless. Sending the sms will either get you nothing or if you get something it will most likely be a big piece of junk taking away precious space in your house. A winner's curse!

The possibility of suffering such a winner's curse (WC) may induce some agents to ignore their private information and pass on opportunities that come in limited supply. They do so without actually observing others' actions. The simple knowledge that others may have priority over oneself allows for the necessary inferences. Agents' behavior in such environments leads to theoretical considerations that we explore in this paper. We offer insights that are relevant to market design.

Crowdfunding markets[1] present environments such as the one in our model. For example, profounder.com is one of many wesites that provide a platform for entrepreneurs to obtain funding from micro-investors. There is a limit to the total number of individuals that may finally invest.[2] The entrepreneur obviously

[1] According to the MacMillan Open Dictionary, crowdfunding is the use of the web or another online tool to get a group of people to finance a particular project. The American Jobs Act, from the current US administration, includes plans to work with the SEC to review securities regulations in ways that expand crowdfunding opportunities.

[2] The limit is imposed by US law. But one can imagine that even without legal limits, and given an average size of investment, there is a economic limit to the initial funding any entrepreneur

wants to maximize the number of potential investors. But the existence of the limit can give rise to the WC reasoning of the "robot" example: a potential investor may argue that if, in spite of the limit in the number of investors, he becomes one of them then it is because others choose not to invest. If they do so because, according to their information, the project is not worth it, then maybe it is better for him not to invest. If more investors argue the same way, then demand for investing in the project may turn out to be low, contrary to the entrepreneur's desires.

We model this situation as a simultaneous choice game where the WC effect is internalized at the same time by all decision makers. In particular, our model considers a set of agents that face the opportunity to invest in a project. There is a limit to the total number of agents that may invest. If the number of agents that choose to invest exceeds the number of slots in the project, then these are assigned according to an exogenous priority order. Agents do not observe the actions of others. Thus when deciding to invest or not they don't know whether agents with a higher priority have invested or not and thus whether there is any available slot for them. Specifically, one can imagine the situation as one in which agents (investors) stand in a line and decide simultaneously whether or not to invest. The decision is taken without knowing what other agents choose to do. After decisions are made, the planner (entrepreneur) goes to the first agent in line and asks him for his decision. She assigns him a slot in the investment if he chose to invest and moves on to the next in line. The process continues until all agents have been asked or no more slots are available. Payoffs depend on whether or not an agent is assigned an investment slot. They further depend on an unknown state of nature which determines the returns of the investment. In a "good" state investing gives a high payoff, while in a "bad" state it is better not to invest. Each agent also has some private information concerning the state. This comes in the form of a binary noisy signal which points to a good or a bad state.

The main ingredients for our model are incomplete information, a common value and the limited supply of investment opportunities. The latter makes other investors' decisions relevant for everybody else, or, in particular, for those that follow in the line. Without limited supply, the problem becomes a sum of individual decision problems, independent of each other, since inferences about others' behavior are unnecessary. Only when supply is limited can one argue that being able to invest means that others with a higher priority have

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not done so. Incompleteness of information and the common value turn this argument to the WC argument described above. Awareness of the WC drives equilibrium behavior in our model: individuals in the front of the line decide according to their private information; the ones that stand further back, ignore their private information and simply do not invest. This is because for the ones standing in front, whether they get a slot does not depend on what others do. The ones in the back can only obtain a slot if the ones in front choose not to invest. This restriction of the market allows them to make equilibrium inferences about the private information of the agents that stand in front of them in the line. If several agents in front choose not to invest it must be that their private information points to the "bad" state. In that case it might be better not to invest, even if one's own private signal points to a good state. Thus individual behavior in equilibrium depends on one's position in the line.

Once we understand individual behavior in our model we can see how other factors may affect the equilibrium in such a market. Anything that can affect the strength of the WC can have an impact. For instance the number of available slots: the WC argument's strength is different in the case of only one available slot compared to the case of 50 slots. For an agent in position 51 obtaining the single available slot is almost certainly a consequence of the investment opportunity being bad. In the second case, even when the state is good, it is enough for a single agent of the 50 preceding in the line to get a wrong signal for a slot to be available. Another interesting issue is the knowledge an individual has about his position in the line. This may not always be perfect and it has an impact on the number of agents that choose to play informatively (follow their signal) or herd (ignore their signal).

Notice that we assume no complementarities among investors' actions. Whether others invest or not does not affect the quality of the investment. It may simply reveal their private information. Thus, we have that factors such as the knowledge about the priority order and the size of the supply of investment opportunities, both unrelated to the quality of the investment and the investor's payoff from it, become determinant for the demand for the investment slots.

In the base-line model, we assume that agents know the exogenous priority order, that is each agent knows exactly his position in line. After a detailed analysis of this case we consider an alternative scenario where priorities are determined by a lottery. The realization of this lottery takes place after investment decisions are made. This scenario represents the other extreme: agents have no knowledge of where they stand in line. We also consider an intermediate case with a Bernoulli arrival process that generates a random assignment. In each period an agent arrives with a given probability. Each agent is aware of this process but does not know how many other agents have arrived before him. Still,the date of arrival gives him some idea about the distribution of this number that allows him to build an expectation of the probability of getting a slot.

We fully characterize equilibrium behavior in our model. When the position in the line is known, agents in the front choose whether or not to invest according to their private information. The ones further back ignore their private information and choose not to invest. Equilibrium in the Bernoulli arrival process model shares the same features. When the priority is set by a lottery, all agents choose not to invest with positive probability even when their private information indicates they should do so. Increasing the available slots affects agents differently, depending on their position in line. When an agent's position in line is greater than the available slots, but close to that, the increase in the number of slots reduces the WC effect and makes investing more attractive. The contrary is true for agents further back in line. With high uncertainty about priority the final direction of the effect depends on the specific parameters.

1.1 Literature Review

Rock (1986) (29) studies the market for IPO's which is an example of a market with incomplete information and limited supply. He uses a "lemons market" type of model to explain the underpricing of initial public offerings (IPO's). In his model uninformed investors compete with informed ones. The first face a winner's curse since they know they can invest only if informed investors consider the offering price too high with respect to the expected market price. The issuer must therefore underprice in order to attract the uninformed investors. A significant body of empirical literature has followed, trying to verify this explanation of IPO underpricing (see Ljungvist, 2007 (22), for a survey). In our model we obtain the winner's curse is of a different nature. There is no asymmetry in information between agents. We show that it is the market design features that determine the strength of the curse. Our model does not share the aim of explaining IPO underpricing. Still, our results suggest that if such underpricing is due to a winner's curse effect, any empirical strategy trying to identify such effect must take into account the institutional settings of the market studied and possibly take advantage of any variation in these.

The paper by Thomas (2011) (32) shares with us the interest in studying markets with limited availability of different goods and incomplete information. However, there are several differences both in the approach and the kind of results obtained. Her paper examines the situation in which different agents acquire information about different alternatives through experimentation. The fact that some of these are limited in supply gives rise to strategic interactions when agents decide on the duration of experimentation. Still, agents here do not learn from one another, whether by observation or in equilibrium. The strategic incentives for choosing one alternative are of a preemptive nature. In our case it is equilibrium beliefs that push an agent to choose something contrary to his information. Furthermore, in Thomas' paper the priority over choices is endogenous. Agents decide when to stop experimenting and grabbing an option. We focus on exogenous priorities.

Given that the marketplace we study does not involve prices, the literature on matching markets is a natural place to look for parallelisms. One approach to matching markets looks at specific matching games. Perhaps the first attempt of such an approach has been the work of Becker (1973) (6). Within this literature and more recently, some papers have considered, as we do, environments with incomplete information and a common value. In particular, a paper that is closer to our work and that is the first to identify the type of winner's curse that influences behavior in our model is the one by Lee (2009) (20). He looks at the decentralized college admissions market and finds a rationalization for the "early admissions" system on the basis of this curse. We focus on a centralized market and see how the use of a matching mechanism can create the curse. This induces herding on the part of some participants in order to avoid it. Another example is Chade (2006) (9). He looks at a decentralized marriage market and detects what he calls the acceptance curse. A participant can infer information by the event of being accepted by a partner at a given point in time. This acceptance may mean that one's value is higher than what one thought about oneself. This is different from the curse in our model where the information generating the curse comes from the equilibrium play of competing agents and concerns the value of the chosen alternative, and not one's own value.

Generalized matching markets with incomplete information were first studied by Roth (1989) (30), and the literature remains active (see for example Ehlers and Massó, 2007 (13); ,Pais and Pinter, 2008 (28)). Incompleteness in these examples concerns knowledge about others' preferences on the part of a participant in the matching market. This literature is interested in understanding the stability and strategy-proofness of matching mechanisms. Chakraborty et al. (2010) (10), follows this line of research and introduces the additional element of value interdependency among participants.

We study how in the presence of incomplete information and a common value agents can make inferences about others' information in equilibrium and the effect of such strategic considerations on the market's performance. In our model it is the assignment mechanism that is used to resolve the problem of limited supply that allows for such inferences. Milgrom and Weber (1982) (25), McAfee and McMillan (1987) (24) study similar effects that arise in auctions. Outside the realm of markets, Austen-Smith and Banks (1996) (4) and Feddersen and Pesendorfer (1997)(15) first studied the implications of such strategic considerations in voting and collective decision making.

The idea that rational individuals may take decisions ignoring their private information is not new. We have just mentioned the case of strategic voting, but probably the most prominent case is the one of social learning and informational cascades (Banerjee, 1992 (5);, Bikhchandani et al., 1992 (7)). This literature studies the case where individuals with imperfect information and a common value move sequentially and can observe the actions of some or all predecessors before making a decision. Gale and Kariv (2003) (16) and Acemoglu et al. (2008) (1), study the case where agents learn through their social network. An informational cascade starts when an individual ignores his private information because the information inferred by observing others' actions points to the other direction. Since his action conveys no new information, all individuals following him act in the same way. Herding behavior does not occur if actions of others were not observed. In Callander and Hörner (2009)(8), for instance, the exact actions of others' are not observed, but only the aggregate choices. Herding in these cases ceases to be an equilibrium feature. If actions are taken simultaneously agents should follow their information. But not if there is limited capacity. This is what happens in our paper and what builds a bridge with the informational cascade model. The general environment is the same but in our case actions are simultaneous and one of the two choices has limited capacity. For a general overview of the literature on social learning in markets the reader should look at the books by Chamley (2004) (11) and Vives (2010) (33).

Agents in our model are fully rational. It is not clear whether this is the right assumption in such a model, since different approaches find experimental and empirical evidence point to different directions. On the one hand, there is evidence that individuals are sophisticated enough to infer information from others' actions triggering informational cascades (Anderson and Holt, 1997 (3); Hung and Plott, 2001 (18); Alevy et al., 2007 (2); Goeree et al., 2007 (17)). On the other hand, evidence points to the opposite direction with respect to sophistication and its relation to the winner's curse. Both in the lab and the real world the majority of individuals fail to take the WC into account (Kagel and Levin, 1986(19); Lind and Plott, 1991 (21)). Our simple model provides a framework in which both situations can be tested. We use it in a related paper (Louis, 2011(23)) to test whether the same individuals are sophisticated enough to follow herds, but not so sophisticated as to avoid the winner's curse. This type of behavior is not predicted by the salient theories of play for games with incomplete information and a common value, such as "level-k reasoning" (Stahl and Wilson, 1995 (31); Nagel, 1995 (27); Crawford and Irriberi, 2007 (12) or "cursed equilibrium" (Eyster and Rabin, 2005 (14)).

2 The model

Agents. There are $n \geq 2$ agents that must choose whether or not to invest in an investment opportunity presented to them. Let $x_i \in X = \{I, O\}$ denote the choice of agent $i \in N = \{1, ..., n\}$. There are only $k < n$ available slots in the investment. This means, it is not possible for all agents to invest. Whether an agent is assigned to one of the available slots is determined by a mechanism $f : \{I, O\}^n \rightarrow \{I, O\}^n$. The assignment follows an exogenous priority order. An agents index denotes the agent's priority: agent i has priority over agent j if $i < j$. Let $f_i : \{I, O\} \times \{I, O\}^{N-1} \rightarrow \{I, O\}$ denote the outcome of the assignment for agent i given his and others' choices. The following holds:

$$f_i(x_i = I, x_{-i}) = \begin{cases} I & if \ \left|\{x_j = I\,,\ j < i\}\right| < k \\ O & ,\ otherwise \end{cases}$$

$$f_i(x_i = O, x_{-i}) = O$$

Information. The state of nature is $\theta \in \Theta = \{G, B\}$. Agents have a uniform *common prior* about the state of nature. This means that the a priori probability of θ taking either value is $\frac{1}{2}$. [3] Before making a choice, each agent receives a noisy private signal $s_i \in S = \{g, b\}$ about the state of nature. Private signals are

[3]Considering non-uniform priors is also possible. Since it does not affect results in a particularly interesting way we choose not to do so, in favor of expositional clarity.

independent conditional on the state of nature. The following table indicates the probability of the signal taking either value conditional on the state θ.

		θ: G	θ: B
s_i	g	q_G	$1 - q_B$
	b	$1 - q_G$	q_B

Payoffs: An agent's utility function has the following form:

$$u_i\left(f(x_i, x_{-i}), \theta\right) = \begin{cases} 1, & f(x_i, x_{-i}) = I \ \ and \ \ \theta = G \\ 0, & f(x_i, x_{-i}) = I \ \ and \ \ \theta = B \\ \gamma, & f(x_i, x_{-i}) = O \end{cases}$$

with $0 < \gamma < 1$. In other words, the payoff of an agent that obtains a slot in the investment is normalized to 1 if the state is "good" and 0 if the state is "bad". When an agent chooses not to invest or does not obtain a slot, he gets $\gamma \in (0, 1)$.

We can view this parameter as the value of a safe outside option. In the case of micro-investors it could be the return one gets by keeping the money in the bank. This being the same for agents that directly choose not to invest and for the ones not obtaining a slot implies that there is no cost from choosing to invest. This may not be true in some occasions. For instance, participating in an IPO may involve non-negligible transaction costs that are independent of whether or not one obtains shares of the company in the end. Adding cost for investing in our model is possible and mathematically tractable. Nevertheless, it will become clear further on that including such costs here would only reinforce our results about agents behavior in such a market. Hence, not including them makes both our results stronger and the exposition cleaner.

Coming back to the image of agents standing in a line, one can view the model we have described in the following way. An agent's index denotes his position in the line. Given the limited availability of investment slots, deciding to invest does not guarantee the agent a slot. The assignment mechanism works in a way that an agent that chooses to invest obtains a slot only if less than k agents standing in front of him, to invest. If an agent's position (index) is less than k than obtaining a slot only depends on his own decision.

In real markets, one's position in the line might depend on one's time of arrival when a the "first-come, first-served" method is used. Or it might depend on some priority assigned by the seller or planner. In the micro-investment example for instance, the entrepreneur might want to give priority to close friends and family over other investors. In a market for "public protection" housing there might be social criteria that determine the priority of potential buyers.

For the moment we assume that each agent knows exactly his position in the line. This assumption can be strong and we later relax it in different ways. Still is useful to start of this way for two reasons. On one hand it allows for a better understanding of the forces that determine equilibrium behavior. On the other hand it is an important building block in the calculation of equilibria in the other environments we explore later on.

As was mentioned in the introduction, agents decide whether or not to invest without observing what others choose to do. There is neither communication among agents nor the possibility for social learning by observing other actions. In some environments this makes sense. For instance in IPO's, investors must decide whether or not to participate in a simultaneous fashion. The stronger argument for this assumption will be clear once we present our results. As we shall see, the equilibrium behavior of agents in our model shares characteristics with the behavior of agents in models of social learning. In particular the fact that some agents ignore their private information and choose a particular action. By obtaining these results with agents acting simultaneously we show how this behavior can emerge in such an environment and what factors drive it.

We will further assume that the following condition holds:

Condition 1.

$$\frac{1-q_G}{q_B} < \frac{\gamma}{1-\gamma} < \frac{q_G}{1-q_B}. \tag{1}$$

This condition makes the problem interesting. It makes sure that when an agent has no further information than his own private signal, his best response depends on the signal's content. A signal $s_i = g$ indicates that investing is a "good" choice. A signal $s_i = b$ indicates it is better not to invest. As will become clear further on, were this not true, *all* agents would choose to invest (for low γ) or not to invest (for high γ) independently of their signal.

Up to now we have defined a set of agents that can take actions out of a particular set and have a particular type which is given by their private information. Their actions lead to payoffs that depend on the state of nature and on the actions of other agents. The environment is further defined by the available slots, the value of the outside option and the precision of the private information. All these define a bayesian game $\mathcal{G} = \langle N, \Theta, \{X, S, u_i, \}_{i\in N}, k, \gamma, q_G, q_B\rangle$. The relevant concept that we use to solve such a game is the one of *Bayesian Nash equilibrium*. In our specific context this equilibrium refers to a strategy for each agent that describes the action the agent takes depending on his private information: $x_i^* : S \to X$. The strategy must be such that and that it maximizes his expected payoff from the game given all other players' strategies: $E[u_i(x_i^*, x_{-i}^*)] \geq E[u_i(x_i^{'}, x_{-i}^*)], \forall i \in N$, and given his beliefs about others' private information.

3 Equilibrium behavior

As was mentioned, agents in our model can neither communicate or observe each others' actions. If there was no limit in the number of available slots, or simply $k \geq n$, then each agent would obtain a slot if he chose to invest. Our model would reduce to a sum of n individual decision problems in which each agent would choose according to his signal. Restricting the supply of slots forces agents to make strategic considerations when making their decision. In particular, agents standing at positions beyond k realize that they can obtain a slot only if less than k of the preceding agents choose not to invest.

We now use the simplest possible example to demonstrate how such strategic considerations affect agents' behavior in such a game.

Example 1

In this example we consider only two agents: $i \in N = \{1, 2\}$. The capacity limit is the lowest possible: $k = 1$. Let us also assume that $q_G = 1$. Condition 1 then reduces to $q_B > 1 - \frac{1-\gamma}{\gamma}$ and we assume this holds. Notice that with this choice of parameters for the signal accuracy, if a player observes a signal $s_i = b$ he knows that the state of nature is $\theta = B$ with probability 1. This is because there is zero probability of obtaining such a signal when the state is $\theta = G$. Agent 1 stands in line in front of agent 2, or in other words, he has got priority over agent 2. This means that agent 2 can obtain the slot only if agent 1 chooses not to invest. For agent 1 the outcome depends only on his own choices.

First consider agent 1. He is the first in line. Whether he obtains a slot depends only on his choice. Since Condition 1 holds, his decision depends on his private signal. If $s_1 = g$ he chooses to invest: $x_1(g)^* = I$. If $s_1 = b$, then he chooses not to invest: $x_1^*(b) = O$.

Agent 2 is second in line. He chooses simultaneously with agent 1. Therefore, even if he chooses to invest he does not know whether or not he will obtain the single slot. This depends on agent 1's choice. If agent 1 chooses not to invest then agent 2 can obtain the slot if he chooses to invest. If agent 1 chooses to invest, then there is no slot available for agent 2 and he gets the outside option. Still, he knows that agent 1's decision depends on the private signal s_1. He also knows that his own decision only matters when agent 1 chooses not to invest. He must therefore decide conditioning on this event. But agent 1 chooses not to invest only when he observes $s_1 = b$ and this is only possible when $\theta = B$. Thus agent 2 knows that his decision matters only when the state is "bad" and in that case he should not invest. Notice that this does not depend on s_2, the signal observed by agent 2. Therefore, agent 2 decides not to invest, independently of his private signal: $x_2^*(s_2) = O$.

The two agents in this example end up playing very distinct strategies in equilibrium. The first agent follows his signal, while the second agent ignores it and chooses not to invest. From now on we shall refer to the strategy of agent 1 as informative play and to to the strategy of agent 2 as herding.

Informative play: *The strategy in which an agent i chooses according to his signal:*

$$x_i(g) = I\,,\ x_i(b) = O$$

Herding: *The strategy in which an agent ignores his private signal and does not invest:*

$$x_i(g) = x_i(b) = O$$

First of all one should note that the reasoning that leads agent 2 to choose such a strategy is based entirely on the fact that the number of slots is limited. Were this not the case it would not be possible to make any inferences about agent 1's actions and information.

The second point to notice is that the behavior of both agents would be the same in equilibrium if there were more agents standing behind them in the line. What is more, it is easy to see that any agent standing behind agent 2 would also herd in equilibrium. This is because, since agent 2 is herding he does not affect any other agents. Thus the hypothetical agent 3 faces the exact same situation as agent 2 and also chooses to herd. The same would be true for any other agent standing in line after them.

This simple example demonstrates the main feature of equilibrium in such games. Agents standing in the first positions of the line play informatively.

After some point in the line agents switch their equilibrium strategy to herding. The point where the switch takes place lies at a position grater than the number of available slots. The following proposition formalizes this result.

The result of the example is generalized in the following proposition.

Proposition 1. *Consider a game* $\mathcal{G} = \langle N, \Theta, \{X_i, S_i, u_i,\}_{i\in N}, k, \gamma, q_G, q_B\rangle$ *and assume Condition 1 holds. There is a unique Bayesian Nash equilibrium in this game. In equilibrium, all agents with index* $i < \hat{m}(k, \gamma, q_G, q_B)$ *play informatively. All others herd and choose* $x_i = O$*, independently of their signal. Furthermore,* $\hat{m}(k, q_A, q_B) > k$*.*

Proof. *All proofs can be found in the appendix.* □

What drives this result is the same as in the two-agent example. Agents with an index higher than k know that they can obtain a payoff higher than their outside option only if the state is "good" and less than k agents of the ones in front of them choose to invest. But given that agents in the front of the line play informatively, conditioning on the event that less than k agents choose to invest (which means that less than k agents received a signal $s_i = g$) reduces the probability of the state being good. There is an increased probability of obtaining a slot in the "bad" state. This is the winner's curse effect. This effect becomes stronger the further back one stands in the line. Therefore, eventually agents switch away from informative play as we move towards the back, in order to avoid the winner's curse.

One important feature of this result is that a significant number of agents never choose to invest. This means that with positive probability less than k agents invest and obtain a slot, even when the state of nature is "good". This ex-post inefficiency is reminiscent of the same inefficiency encountered in the social learning model. We study that further on when we make a comparison between the two different models: our own and a social learning model, where agents decide sequentially, with a limited availability of investment slots.

For now we must point out that the equilibrium is efficient. The number of agents playing informatively maximizes the sum expected payoffs. This is stated in the following proposition.

Proposition 2. *Given* k, γ, q_G, q_B *that satisfy condition 1, the unique equilibrium strategy profile of a game* $\mathcal{G} = \langle N, \Theta, \{X_i, S_i, u_i,\}_{i\in N}, k, \gamma, q_G, q_B\rangle$ *with known priorities, for any N, is ex ante efficient. Another pure strategy profile of the game is ex ante efficient if and only if the same number of agents play informatively as in the equilibrium profile.*

The reason why this holds is simple. The number of agents playing informatively is such that any agent that plays informatively has an expected payoff higher than what he obtains by herding, which is the outside option. If less agents play informatively, then they are forgoing to possibility of a higher expected payoff. If more agents play informatively, then some have an expected payoff smaller that their outside option. Both these cases result in a smaller sum of expected utilities and are therefore inefficient.

Another feature of the equilibrium to note is the sorting of agents and strategies. Low index agents play informatively while high index agents herd. This means that what to an external observer might seem as some sort of correlation between priorities and preferences or information is simply rational equilibrium behavior of agents with identical preferences.

3.1 Comparative statics.

To get a better grasp of how equilibrium behavior depends on the various parameters of the model we perform comparative statics. It is important to understand what exactly is “moving” when we change one of the parameters. For that one has to understand the mechanism that underlies proposition 1.

As long as Condition 1 holds, agents that receive a signal $s_i = b$ never choose to invest. The ones that receive $s_i = g$ calculate their expected payoff from investing, taking into account the fact that to obtain a slot it must be that less than k agents in front of them invest. They compare this to the payoff from the outside option γ. Whether an agent plays informatively or herds depends on this comparison. Thus any effect of a change in parameters on equilibrium behavior must come through the effect the change has on the expected payoff from investing after observing $s_i = g$. This is given by the following function in which we assume all agent in front of i play informatively:

$$E[u_i(I,g)] = Pr(G|g)\Bigg[\underbrace{\overbrace{Pr\Big(\big|\{s_j=g,\ j\le k\}\big|<k\,|\,G\Big)\cdot 1}^{payoff.\ when\ a\ slot\ is\ free} + \overbrace{Pr\Big(\big|\{s_j=g,\ j\le k\}\big|\ge k\,|\,G\Big)\gamma}^{payoff\ when\ no\ free\ slot}}_{payoff\ when\ state\ is\ "good"}\Bigg]$$
$$+ Pr(B|g)\Bigg[\underbrace{\overbrace{Pr\Big(\big|\{s_j=g,\ j\le k\}\big|<k\,|\,B\Big)\cdot 0}^{payoff.\ when\ a\ slot\ is\ free:\ WC} + \overbrace{Pr\Big(\big|\{s_j=g,\ j\le k\}\big|\ge k\,|\,B\Big)\gamma}^{payoff\ when\ no\ free\ slot}}_{payoff\ when\ state\ is\ "bad"}\Bigg] \tag{2}$$

The first term of the second bracket in the RHS represents the winner's curse. It is the payoff an agent receives when investing and obtaining a slot when the state is "bad". The number of agents that receive a particular signal given the state follows a binomial distribution. Hence the probability of less than k agents to have received a signal $s_j = g$ given the state, is given from the cumulative density function (cdf) of the binomial distribution with the appropriate parameters. Let $F_{(m,G)}(k)$ represent the cdf of $Bin(m, q_g)$ and $F_{(m,B)}(k)$ represent the cdf of $Bin(m, 1-q_B)$. Thus we have:

$$E[u_i(I,g)] = \frac{q_G}{q_G+1-q_B}\Big[F_{(i-1,G)}(k-1) + \Big(1-F_{(i-1,G)}(k-1)\Big)\gamma\Big] + \frac{1-q_B}{q_G+1-q_B}\Big(1-F_{(i-1,B)}(k-1)\Big)\gamma \tag{3}$$

The equilibrium behavior of a particular agent is determined by whether this expression is above or below γ, the value of the outside option. When it is above, the agent invests. When it is below he herds.

3.1.1 The value of the outside option.

The value of expression 3 is increasing in γ. Still, the sum of the factors of γ is lower than 1. This means that as we increase γ 3 also increases but at a slower rate. So let us consider the last agent in line that plays informatively for some low γ. This means that for him $E[u_i(I, g)] > \gamma$. Now suppose we increase the value of the outside option. While both sides of the inequality increase, the RHS does so faster, so eventually it will switch. This agent will change his strategy from informative play to herding.

Here the value of the outside option is given relative to the possible payoffs of the investment that are normalized. These values would normally depend on whoever tries to attract the investors. We do not model such an agent in any form here. Still, what we learn here is that an entrepreneur trying to attract investors, can do so by making the investment more attractive relative to the outside option. This is assuming she has no other information that can be inferred by her choices. This result is similar to the one obtained in Rock (1986) (29), where he concludes that the seller in an IPO might want to underprice in order to attract the uninformed investors. We get a similar conclusion, but here we do not assume any asymmetry in information among investors.

3.1.2 The number of available slots.

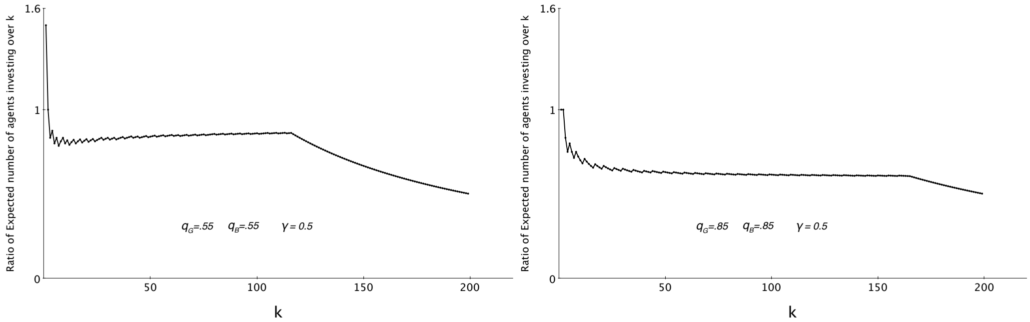
The number of available slots is a parameter which a market designer can control to a significant extent in many markets. For instance an entrepreneur might decide the maximum number of investors she wants to take on board her project, or a department may decide within limits on the number of available openings.

The effect on equilibrium from increasing the number of slots is clear cut: more agents play informatively.

Proposition 3. *Consider a game $\mathcal{G} = \langle N, \Theta, \{X_i, S_i, u_i, \}_{i \in N}, k, \gamma, q_G, q_B \rangle$ and assume Condition 1 holds. Then $\hat{m}(k, q_A, q_B)$ is increasing in k.*

To understand why this happens one must understand that it is the limited supply of slots that gives rise to the winner's curse. Obtaining a slot when the supply is limited happens only when "enough" preceding agents choose not to invest. When k is low, "enough" represents a large number of agents. When k is high, "enough" represents a small number of agents and thus a weaker winner's curse effect.

We must notice here that we obtain this clear-cut result for the case where priorities are known. For the cases we study further on with priority uncertainty this result may not hold, depending on the other parameters.



(a) low accuracy (b) high accuracy

Figure 1: Examples of how the demand for investment evolves with k

Although the number of agents choosing to invest increases with k it is interesting to see the rate of this increase. The following graphs in figure 1 show for two different levels of signal accuracy the ratio of the expected number of agents that choose to invest in equilibrium, over the number of available slots. When this ratio is above one, we expect excess demand. When the ratio is below 1 we expect excess supply. We observe that excess demand only occurs for low levels of k. The ratio drops off fast. This happens because of the effect of increasing k on the winner's curse. How it evolves further depends on the the other parameters. Here we see that for low levels of signal accuracy the ratio show a tendency to increase again, while for low levels of accuracy it continues decreasing. An explanation for that is that when accuracy is high the winner's curse effect remains persistent. More agents play informatively because more agents have an index below k but for agents with a higher index the effect is still there. When accuracy is low, the increase in k has a strong attenuating effect on the winner's curse. Therefore, not only agents with am index below k switch, but also a significant number of agents with a higher index. In both graphs there is a drop of the ratio in the end. This is due to the fact that already all agents are playing informatively after that point. Hence, increasing k has no further effect on equilibrium.

3.1.3 The accuracy of information.

The accuracy of information in our model is represented by the parameters q_G and q_B. The higher these parameters are, the stronger is the signal the agents receive. As we explained, when an agent decides in equilibrium he also takes into account the signals of others that stand in front of him in the line. So suppose an agent receives a “good” signal. The higher the accuracy of the signal, the stronger the indication that the state is actually “good”. But in equilibrium this agent may obtain a slot only if enough of the preceding agents choose not to invest. These agents must have received a “bad” signal. The higher the accuracy of the signals the stronger an indication it is that the state is actually “bad”. Thus, the increase in accuracy has a positive effect through one’s own signal but a negative effect through the signals of preceding agents.

Which effect dominates? This depends on where an agent stands in line. For equilibrium what matters is agent $\hat{m}$. If a change in the accuracy has a positive effect in his expected payoff, he (and maybe more agents) may switch from herding to informative play. If the effect is negative, then it is possible that some agents that played informatively, switch to herding. This would give a new $\hat{m}$ with a lower index.

Changes in q_G. An increase in q_G means that it is more likely to receive a signal $s_i = g$ when the state is “good”. By bayesian logic it also means that having received a such a signal it is more likely that the state is “good” . Looking at expression 3 we can see how this creates the two opposite effects described. On one hand, the factor $\frac{q_G}{q_G+1-q_B}$ increases while the complementary factor $\frac{1-q_B}{q_G+1-q_B}$ decreases. This represents the positive effect from one’s own private signal $s_i = g$ becoming stronger. At the same time though, the term in the brackets decreases, since $F_{(i-1,G)}(k-1)$ is decreasing in q_G. This represents the effect of the “bad” signals of preceding agents becoming stronger.

The graph in figure 2 shows an example of how changing q_G affects the shape of the function in expression 3. The points in the rectangle are the ones corresponding to the threshold agent $\hat{m}$.

Changes in q_B. An increase in q_B means that it is more likely to receive a signal $s_i = b$ when the state is “bad”. Again, by Bayesian logic it follows that having received a signal $s_i = g$ it is more likely that the state is “good”. In expression 3 we can see the two opposite effects. The two fractions move in the same direction as before. Now it is in the term in the last parenthesis where we observe the opposite negative effect. This term decreases.

The graph in figure 3 shows an example of how changing q_B affects the shape of the function in expression 3. The points in the rectangle are the ones

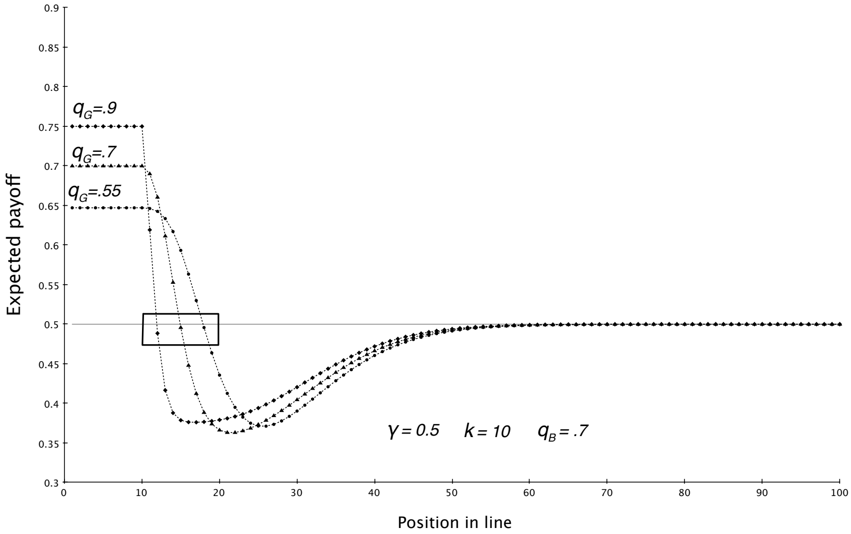


Figure 2: Comparative static with respect to q_G.

corresponding to the threshold agent $\hat{m}$. Note here that $\hat{m}$ moves to the opposite direction than when we were changing q_G.

4 Simultaneous play vs. Social learning.

In our model agents do not learn from each other. There is no communication between them, nor is it possible to observe each others' actions. Yet, the behavior we observe in equilibrium resembles the one found in models with social learning in which agents take actions sequentially and can observe what others do (Banerjee, 1992 (5);, Bikhchandani et al., 1992 (7)). In this section we compare behavior in our model with the one in such a model. The social learning we consider follows the exact same setup as our model with one difference: agents take actions sequentially and observe the actions of the agents standing in front of them in the line. This is equivalent to adding a limited number of slots for one of the alternatives in the binary model in Bikhchandani et al. (1992). In the sequential model, the limited number of slots does not affect strategic behavior. Since agents observe the actions of others they can accurately their private information when making their own decision. If the slots are filled the game ends and remaining agents obtain their outside option. The interesting equilibrium feature in such a model is the possibility of an informational cascade emerging.

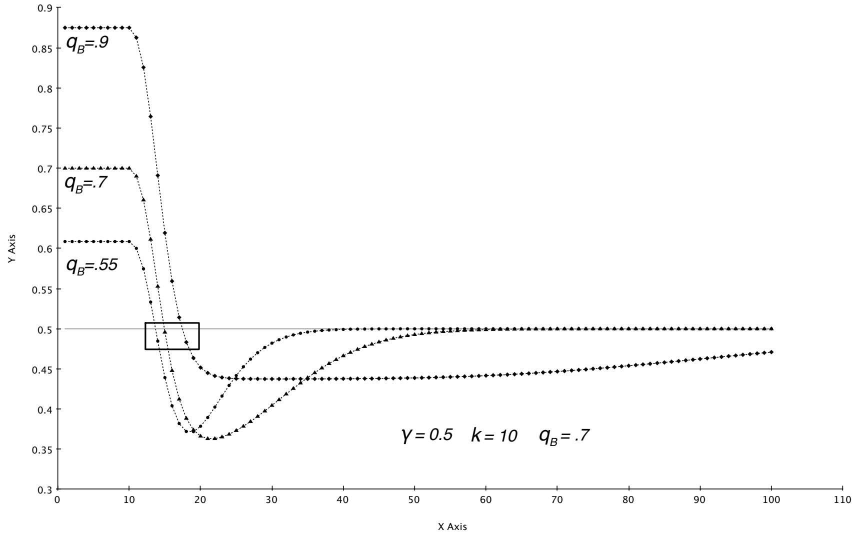


Figure 3: Comparative static with respect to q_B.

After observing a particular sequence of actions an agent's beliefs about the state may be such that his private signal does not make a difference about the optimal action. In this case the agent herds (ignores his private information) and so do all agents after him in the line. Informational cascades can go either way with agents herding choosing to invest or not to do so. There is also the possibility of agents herding on the wrong decision.

The equilibrium outcome in the two models can be very similar. For instance, in the two gent model described in example 1, allowing the second agent to observe the action of the first agent, makes no difference in the outcome observed in equilibrium. The first agent may invest or not, depending on his signal, while the second agent always obtains his outside option. This happens because the equilibrium inferences made by the second agent in the simultaneous game mirror exactly the inferences he makes in the sequential game.

Such similarities persist in games with more players and different levels of k when the value of the outside option is low. Outcomes change when this value is high. We explain the intuition behind this phenomenon and use numerical simulations to demonstrate the result.

There are two types of mistakes agents can make: not investing when the state is "good" or investing when the state is "bad". The first type is costly when γ is low. That is when the outside option give a low payoff compared to that of a good investment. The second type is costly when γ is high.

In the simultaneous model, informational cascades serve as a mechanism to protect agents from these mistakes. By observing others, agents are able to make decisions based on more information than only their private signal. The "cost" of such a defense mechanism is that sometimes it produces "bad cascades", in which agents all herd on the wrong decision. Still, the probability of such a cascade is relatively low.

In the sequential model, there is again a low risk of committing the first mistake. In equilibrium a large number of agents plays informatively. For the ones that herd, choosing not to invest makes a difference only if the agents playing informatively leave free slots. But this rarely happens when the state is "good". Concerning the second type of mistake, investing in a "bad" state, the agents in the back of the line that herd are protected. Still, agents in the front of the line must rely solely on their private information and it is possible for them to make such a mistake. More so than agents in the sequential model that decide based not only on a single private signal.

So it turns out that what cause a difference in the outcome of the two models is the degree to which agents commit the mistake of investing in a "bad" project. When γ is low, such a mistake is not very costly and furthermore, "bad cascades" in that direction are not very likely in the sequential model. Therefore the outcomes of the models do not vary significantly. When γ is high, such a mistake becomes costly. "Good cascades" protect agents in the sequential model. In the simultaneous model agents commit this mistake more often.

From an efficiency point of view, when γ is high, the sequential game produces better outcomes. For a low γ outcomes do not differ much. In the simulations we perform, efficiency is slightly better in the simultaneous game for low a low k and slightly worst for higher k. Still, differences are of a very small magnitude.

From the point of view of demand, when γ is high there is a higher demand for investment in the simultaneous game, except for very low levels of k. For low γ again demand is higher in the sequential game, but only for very low levels of k is the difference significant.

The following graphs show the results of Monte-Carlo simulations performed in order to compare the outcomes of the two models. For these simulations we produce a vector of private signals. We calculate the equilibrium corresponding to this vector for each model for different levels of k. We repeat the process 10,000 times and take averages of our results. The parameters used in the simulations presented here are $n = 100$ and $q_G = q_B = q = 0.85$. We do the calculations for three different levels of the value of the outside option: $\gamma \in \{.4, .5, .6\}$. The first graph shows the difference in the total welfare (normalized to lie between zero and one) between the two models. Positive values indicate a higher welfare in the simultaneous model. The second graph shows the difference in demand for investment between the two models. Demand here is calculated as the fraction of slots filled in equilibrium. Positive values indicate a higher demand in the simultaneous model.

One can see in the graphs how the differences between the two models become pronounced when γ is high. The kind on the right side of both graphs is due to the fact that once k is high enough all agents play informatively in our model. Therefore increasing k further does not change the equilibrium behavior of agents. Still, it affects the normalized values of welfare and demand.

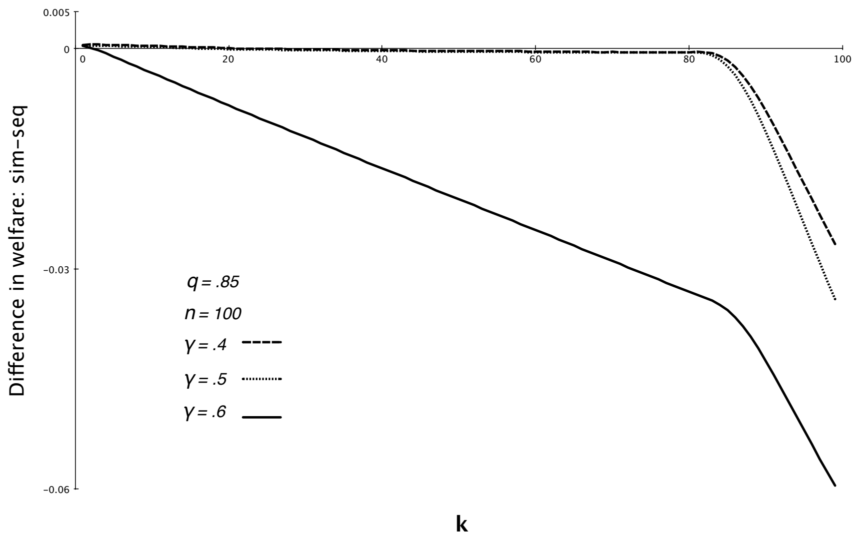


Figure 4: Difference in welfare in the two models.

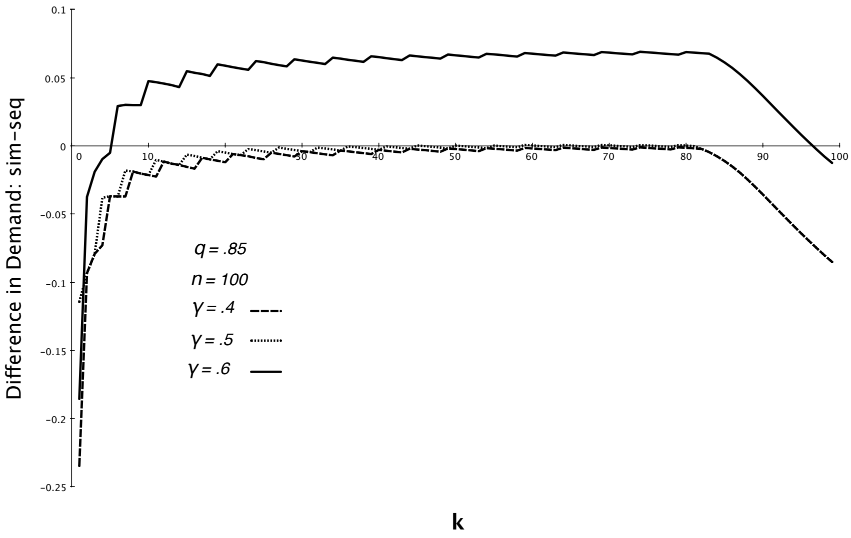


Figure 5: Difference in demand in the two models.

5 Priorities assigned by a lottery.

Up to this point we considered that each agent knew exactly his position in line. We now relax this assumption. In this section we consider the case where a lottery is used to determine the position in line of each agent. The lottery takes place after each agent makes his decision about whether or not to invest.

It makes sense to consider such a variation to our model for two reasons. First, it comes closer to some real life situations where such a mechanism is used, like some IPO's. In general, one could consider this as the other end of the spectrum of possibilities about what agents know about their priority. In reality, different cases might lie anywhere between the two extremes.

The second reason to consider this variation is a theoretical motivation. Notice that now all agents are ex-ante identical. Once they receive their private signal they are differentiated, but even at that point, all agents who observe the same private signal have exactly the same information and available choices. As we shall see, for some range of parameters there exists a symmetric equilibrium in which agents herd with a positive probability. This result highlights the fact that it is the institutional design of the market and not the heterogeneity of agents that give rise to the winner's curse effect. This is important for anybody looking at market data trying to identify such an effect. For instance in the "IPO underpricing" literature in finance the WC effect was described by Rock (1986) (29) but attributed to the existence of differentially informed agents. Empirical strategies trying to verify the theory relied on the existence of such heterogeneous groups. Our result suggests that the WC effect should be present even without differences in information between groups of agents.

From a technical point of view, the introduction of a lottery gives rise to multiple equilibria. Given that now agents are symmetric, we find it reasonable to focus on symmetric equilibria. It turns out there is a unique symmetric equilibrium in mixed strategies. We will denote the game with a lottery as $\mathcal{L}$. Let $L = \{1, ..., n\}$ denote the set of positions in line to which agents are assigned by the lottery.

Proposition 4. *Consider the game* $\mathcal{L} = \langle N, L, \Theta, \{X_i, S_i, u_i, \}_{i \in N}, k, \gamma, q_G, q_B \rangle$ *There exists a unique symmetric equilibrium in mixed strategies in the game with lottery determined priorities. For k sufficiently low and* γ *sufficiently high agents decide to herd with a positive probability.*

To understand where this result comes from one can think the following. If everybody else herds, then an agent knows that he can obtain a slot by choosing to invest. As long as he observes a "good" signal, this is a best response independently of the outcome of the lottery. Now as the probability of all other agents playing informatively increases, it becomes more and more likely to be placed in a position in the back of the line with a high probability of more than $\hat{m}$ agents in the positions in front playing informatively. In such a position the expected payoff is less than the outside option. If this probability is too high, then it is best for an agent to switch his strategy to herding. There is some level of this probability where an agent becomes indifferent between informative play and herding. It is easy to see that as the value of the outside option γ increases, this level becomes lower, since herding becomes more attractive. The opposite happens with the number of available slots k. This is because for a higher k there is a higher chance to be positioned through the lottery to one of the front spots in the line where one is immune to the winner's curse.

A natural question that rises is how the introduction of the lottery affect the characteristics of equilibrium. In particular, what effect does it have on herding behavior? While the symmetric equilibrium allowed us to highlight the existence of the winner's curse effect even with homogeneous agents, it does not lend it self for easy comparison to the equilibrium of the case where agents know their position in line. In the following proposition we have a comparison between pure strategy equilibria.

Proposition 5. *In the lottery game there exist pure strategy equilibria in which $\tilde{m}(k, q_A, q_B, N) \leq N$ agents play informatively. Furthermore, more agents play informatively in such an equilibrium than in the unique equilibrium of the game with known priorities: $\tilde{m}(k, q_A, q_B \geq \hat{m}(k, q_A, q_B) - 1$.*

In the game with no lottery any agent after $\hat{m}$ knows that his expected payoff from investing is less than his outside option and therefore herds. In the game with a lottery as long as there are at least $\hat{m}$ other agents playing informatively an agent can be unlucky and be assigned a position in the line after all these $\hat{m}$ or more agents and also get an expected payoff that is lower than his outside option. Still, this is only of the possible outcomes he faces. It is therefore not necessary that he prefers to switch to herding. Thus it is possible for such a profile with more than $\hat{m}$ agents playing informatively to be sustained as an equilibrium.

Combining this result with the one in proposition 2 it is easy to see that such an equilibrium is not efficient. Thus the uncertainty about priorities introduced with the lottery allows for inefficiencies to be introduced due to the fact that equilibria are possible in which the number of agents playing efficiently is higher than the efficient level.

6 A Bernoulli arrival process

In the Previous sections we have looked at two extreme cases concerning agents' knowledge of their position in line. In the first, they are perfectly informed about it while in the second they have no information whatsoever, since it is a lottery that determines it. Given the results obtained for these two cases, a natural question follows. What happens for "intermediate" cases of uncertainty about priority? By an "intermediate" case we mean one in which agents do not know their position with certainty, but still there is some heterogeneity among agents. Some know it is more likely for them to be in the front while others find it more likely to be in the back. How does such uncertainty and heterogeneity affect equilibrium behavior?

Modeling such a situation for a finite number of agents is not a trivial task. For agents' beliefs to be consistent it would require that the $n \times n$ matrix, representing each agents probability distribution for each position in the line, to be a doubly stochastic matrix[4].

We choose here a different approach. We use a Bernoulli arrival process to model an "intermediate" uncertainty case. In particular, we consider that time is divided in discrete intervals. In each time period t an agent arrives with positive probability p, the arrival rate. While the individual agent knows his time of arrival and the arrival rate at the time of making his decision he does not know the realized number of arrivals in the preceding periods. Coming back to the image of the line, one can think that the line exists inside a room. Agents arrive at the room's door and must make a decision before entering. They can not see how many agents have already entered the room before them. Once they make their decision and enter, they stand in line behind the one's that are there already, but cannot change their decision.

It is of course convenient to take time in this model at face value and consider it as a model of cases where "first come, first served" is used to allocate slots. This would of course fit the "robot example" in the introduction as well as many other cases of markets where such a rule is used. An alternative view would be think of time in the model as an metaphor for the information agents have about their priority. The arrival time t could simply represent a private signal for the agent about his priority. The higher this signal, the more likely it is that the agent is actually in the back of the line. In applications this signal together with the arrival rate contains all the (noisy) information agents have about the total number of agents participating in the market and their individual priority.

Note that now the set of agents is not fixed. There is uncertainty about the total number of players in the game. This makes it a game with population uncertainty (Myerson, 1998 (26)). Still it is not a poisson game, which is the population uncertainty game usually studied, but which is not relevant in this context. To our knowledge it is the first instance of such a game in which population uncertainty is modeled by a Bernoulli process. Let us call it a Bernoulli game.

In such a game the set of agents is replaced by the set of types and a distribution over that set. An agent's type is determined by his time of arrival $t \in \mathbb{N} = \{1, ...\}$ and his private signal $s_t \in \{g, b\}$. The set of types is $T = \mathbb{N} \times \{g, b\}$. The distribution over the set of types is given by the arrival rate p and the accuracy of the signals q_G and q_B.

[4]Each row and each column add up to 1. This should be so because both the probabilities over positions for an agent must add up to 1 as the probabilities for any position to be filled by one of the agents

It turns out that this game has a unique equilibrium with similar characteristics to the equilibrium of the game with no uncertainty. The following proposition describes this result.

Proposition 6. *Consider the Bernoulli game $\mathcal{B} = \langle T, \{p, q_G, q_B\}, \Theta, \{X_{(t,s)}, u_{(t,s)}, \}_{(t,s)\in T}, k, \gamma\rangle$ and assume Condition 1 holds. There exists a unique Bayesian Nash equilibrium of this game. In equilibrium, all agents that arrive at $t < \hat{t}(k, q_A, q_B, p)$ choose according to their private signal. All others choose O, independently of their signal. Furthermore, $\hat{t}(k, q_A, q_B, p) \in (\hat{m}(k, q_A, q_B), \infty)$.*

This result has a similar flavor to the one in proposition 1. There it was agents standing after a specific point in line that choose to herd. Here it is agents arriving after a specific point in time. The intuition that drives it is similar. Suppose everybody plays informatively. Agents arriving early see it as highly likely to be in the front of the line and therefore are happy playing informatively. Furthermore, their payoff does not depend on what others that arrive later do. The later an agent arrives, the more likely it is for him to be placed further back in the line. This depends on how many agents have arrived before him. Since all these agents will be playing informatively, we know from our previous results, that when the probability of being placed towards the back becomes high, eventually it is better to switch one's strategy to herding. The same is then true for all agents arriving after that point in time.

Concerning efficiency, we do not provide a formal result, but it is easy to see that this equilibrium is ex ante (before population uncertainty is resolved) efficient. This is for the same reasons as in the game with known priorities. Of course, once population uncertainty gets resolved but before the revelation of the state of nature, the equilibrium will generally not be efficient. It can only be efficient if the realized arrivals before time $\hat{t}$ equal exactly $\hat{m} - 1$. This will generally not be true.

Now that we have characterized behavior in this model of "intermediate" uncertainty about priorities we can look at one final issue. The relationship between this uncertainty and the behavior of agents. Looking at the equilibrium results for the two extreme cases (known priorities, lottery) one might think that there is a monotonic relationship between uncertainty and the incentives to herd. In particular it looks as if higher uncertainty about one's priority attenuates the winner's curse effect and makes informative play more attractive. In what follows we demonstrate by a counterexample that this is not always the case. The uncertainty about one's priority can have an effect on behavior, but the direction is not always the same. It depends on the whole parameter set.

In the following exercise we calculate the expected payoff of agents arriving at different time periods. For each agent we adjust the arrival rate in such a way that the expected number of earlier arrivals remains the same. For instance there can be two agents, one arriving at time t and the other at $t' > t$. Suppose the respective arrival rates are p and p' such that both agents the expected number of earlier arrivals is $p(t-1) = p'(t'-1) = \lambda$. Still, the variance of the distribution of earlier arrivals is different in each case. It must be $p(1-p)(t-1) < p'(1-p')(t'-1)$. One can therefore argue that in the second case the agent faces a higher uncertainty about his position in line. If the conjecture about the monotonic relationship between priority uncertainty and behavior was true, then we should expect that if the second agent plays informatively in the equilibrium of his game then so would the first agent in his respective game. And if the first herds in the equilibrium of his game, then so does the second in the equilibrium of the respective game. We use a numerical example to show this is not the case.

Example 2

The graph shows an example for a particular choice of parameters $k = 4$, $q_G = q_B = .733$, and $\gamma = .5$. Also, In this example we have fixed the expected number of earlier arrivals for each agent to $\lambda = 6.226$. From the properties of the Bernoulli distribution we have $\lambda = tp$. So given the time of arrival of an agent and in order to keep λ constant, we calculate a different arrival rate for each agent t: $p = \frac{\lambda}{t}$. This means that each agent we consider plays a different game. The horizontal axis shows the time of arrival of an agent. The vertical axis shows his expected payoff from deciding to invest after observing a signal $s = g$. That is: $E[u(I;(t,g)]$. An agent arriving at t plays informatively when this is higher than the value of the outside option, γ. The horizontal line in the graphs indicates the value γ. Thus, points above this line correspond to agents that play informatively in the equilibrium of their respective game. Notice that p is decreasing in t. The variance for each agent t is $tp(1-p) = \lambda(1-p)$ which is decreasing in p. This means that the later an agent arrives in this exercise, the higher the variance he faces.

As can be seen in the graph, the monotonicity one might expect given our previous results is not there. Take an agent arriving at $t = 8$ and one arriving at $t = 14$. They both play in games where all parameters are the same except for the

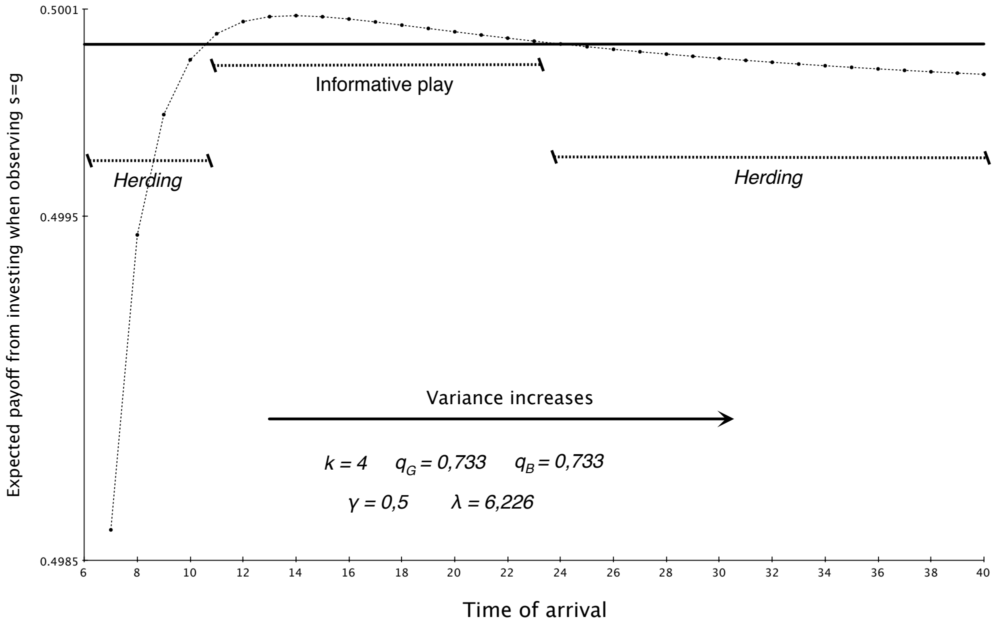


Figure 6: *An example of non-monotonicity in the relationship between uncertainty and behavior.*

arrival rate. Still, this is such that they both expect the same number of agents to have arrived before them. The number of these previous arrivals is a random variable and has the same mean for both agents, but a different variance. This is larger for the agent arriving at $t = 14$. Assuming the agents both observe a private signal $s = g$, their expected payoff from choosing to invest is shown in the vertical axis. It is clear that fro the agent arriving at $t = 8$ it is best not to invest since he obtains a higher payoff from the outside option. The opposite is true for the agent arriving at $t = 14$.

The following graph can help explain this fact. The bell-shaped curves represent the distributions of previous arrivals that agents arriving at t and t' face. These have both the same mean λ. Thus the one for t' is a mean-preserving spread of the one for t. Once this uncertainty is resolved, agents find theme selves in a certain position m in the line. The quasi-U-shaped curve that spans horizontally represents the expected payoff from choosing to invest for an agent in position m that has observed a signal $s = g$. The straight horizontal line indicates the value of the outside option γ. The expected payoff from investing when observing $s = g$ for an agent arriving at t is calculated by taking the sum of the area below $E[u_m(I, g)]$ weighted by the corresponding binomial distribution. For an agent to decide whether or not to invest, this must be compared to the payoff from the outside option. The mean preserving spread for the agent that arrives later, at t', puts less weight close to the mean λ and more weight on the sides of the distribution. While this has a positive effect on the left side where the expected payoff from investing is higher that γ it can have a negative effect on the right side where the expected payoff is less than γ. Which of the effects is stronger depends on the whole parameter set considered.

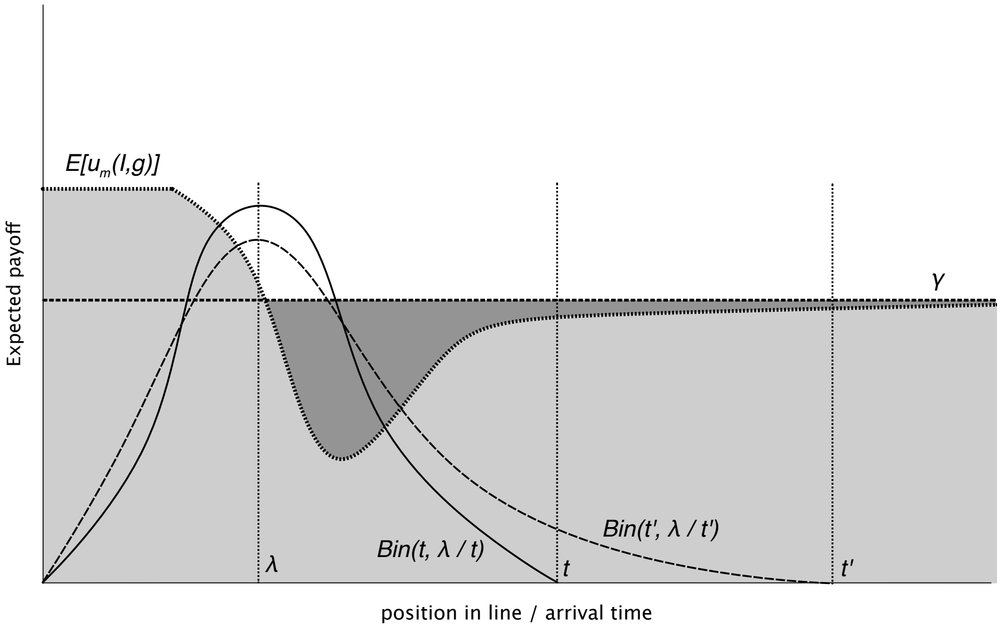


Figure 7: *What happens when the variance increases.*

7 Conclusions

We presented a simple model of a market for *limited* investment opportunities. Incomplete information and a common value, combined with the limited offer of investment opportunities generate a winner's curse effect. Agents' equilibrium behavior depends on their priority, which is exogenous. We discuss how changes in the availability of investment slots, the accuracy of information and knowledge concerning the priority order can have an impact of the demand for the investment opportunities and the performance of the market in general.

In our model, agents face no budget constraint. Furthermore, the supply of investment opportunities is not connected to their payoff. This allows for a more tractable analysis. It is reasonable to think that in reality any change in the supply of investment slots should be connected to a change in the price of investment and its attractiveness. For instance, an entrepreneur seeking up to 30,000 euros of capital for a new project can offer 15 slots for 2,000 euros each, or 20 slots of 1,500 euros each. Increasing the number of slots and maintaining the total capital constant makes each slot more affordable. On the other hand, the returns for each slot will also be lower. In our analysis abstract away from these issues. Nevertheless, our analysis suggests that such a change in the model would not affect the results concerning the existence of the winner's curse and its consequences.

In the literature on IPO underpricing[5], a winner's curse effect is identified as a theoretical possibility but is attributed to a problem of assymetry of information between perfectly informed and completely uninformed agents. Rock (1986) concludes that "...the institutional mechanism for delivering the shares to the public is irrelevant as far as the offer price discount is concerned.". In our paper, we show that institutions matter because a winner's curse can arise even when agents are symmetrically informed. It is the design of the market institutions that determines what the effects of the curse will be. An interesting extension to our model would be to allow for agents to decide whether or not to acquire information before deciding to invest. Given the winner's curse effect, even if the cost of information is low, some agents may decide not to acquire information in equilibrium, giving rise to endogenous information asymmetry. Such a result would form a bridge between our model and the one by Rock. We are currently working on such an extension.

In this paper we find that social learning is not necessary for agents to make inferences about others' information and adapt their behavior accordingly. In environments with incomplete information and a common value, limited supply gives rise to herding behavior. Then, the particular mechanism used to assign priorities determines agents' demand. A natural next step is to think about implications for mechanism design in general. For example, our model can be viewed as a fixed price auction. How does it perform compared to a regular auction? How much information should participants have about others' actions or about their own priority? Building on the basis that we set here, we plan to further explore these issues both theoretically and experimentally.

[5]See the survey by Ljungqvist (2004).

We primarily focused our analysis on the buyers' side of the market. Even so our analysis shows how a seller can influence demand by determining the relative payoffs and the available supply. These conclusions are based on the implicit assumption that the seller is uninformed about the state. If this is not the case, the situation becomes an interesting signaling game in which the seller can reveal information about the state through the choices of available supply and price. Another possible signaling vehicle that is worth exploring is a practice that is observed in some emerging crowdfunding platforms and other markets. The seller there sets a minimum demand threshold that must be covered to make the offer effective. In other words, if the minimum threshold is not reached no money changes hands. It can be interesting to study how a seller would optimally set such a threshold given its signaling content and the presence of the winner's curse. A model in which entrepreneurs use such a threshold to compete for investors is something we view as a potential route for future research.

Our results depend critically on the assumption of fully rational agents, sophisticated enough to be aware of the winner's curse and act accordingly. Whether actual individuals have this level of sophistication is a matter of debate. Experimental and empirical data on common value auctions are not conclusive. Nevertheless, besides contributing to this debate with another open question, our model also provides a useful tool: it represents a simple binary choice model in which the winner's curse appears. Therefore It can easily be used in experiments to test individuals' awareness of the curse or other related issues. Louis (2011) uses the two-agent version of the model from example 1 in such a way. Subjects play the game in the example both sequentially and simultaneously. The question is whether the same individual may be sophisticated enough to detect the course in the sequential game, but not in the simultaneous. It turns out that a significant portion of subjects fall in to this category, something that it not predicted neither by Bayesian-Nash equilibrium, nor by other alternative theories.

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A Proofs

Proof. (**Proposition 1)** The fist k agents in the line face a simple decision problem. Whether they are assigned a slot or not does not depend on what others do. Therefore, given Condition 1 their dominant strategy is to play informatively. This means to follow their signal: $x_i = I$ when $s_i = g$ and $x_i = O$ when $s_i = b$. Any agent standing in position $m' > k$ gets a slot assigned only if less than k of the $m = m' - 1$ agents standing in front of him choose to invest. If this is not the case he obtains γ independently of his decision. He takes this into account when calculating his expected payoff from choosing whether or not to invest.

First let us consider agent $m' = k+1$. and suppose he receives a private signal $s_{m'} = b$. All k agents standing on front of him play informatively and therefore their actions reveal their private signals. In other words, if for example k agents invest, it means that these k agents have received a private signal $s_i = g$. His expected payoff from choosing to invest is:

$$\begin{aligned} E[u_{k+1}(I)|b] =& Pr(G|b)\left[Pr\left(\left|\{s_j = g,\ j \leq k\}\right| < k \,|\, G\right) + \left(1 - Pr\left(\left|\{s_j = g,\ j \leq k\right| < k \,|\, G\right)\right)\gamma\right] \\ &+ Pr(B|b)\left[\left(1 - Pr\left(\left|\{s_j = a,\ j \leq k\}\right| < k \,|\, B\right)\right)\gamma\right] \end{aligned}$$

His expected payoff from choosing not to invest is :

$$E[u_{k+1}(O)|b] = \gamma$$

Note that:

$$Pr\{G|b\} = \frac{1-q_G}{1-q_G+q_B}$$
$$Pr\{B|b\} = \frac{q_B}{1-q_G+q_B}$$
$$Pr\left\{\#\{s_j = g,\ j \le k\} < k \,|\, G\right\} = 1 - q_G^k$$
$$Pr\left\{\#\{s_j = a,\ j \le k\} < k \,|\, B\right\} = (1-q_B)^k$$

We now show that when Condition 1 holds, the expected payoff from investing in this case is always lower than the one from not investing. Suppose not:

$$E[u_{k+1}(I)|b] > E[u_{k+1}(O)|b]$$
$$\frac{1-q_G}{1-q_G+q_B}\left[1-q_G^k+q_G^k\gamma\right] + \frac{q_B}{1-q_G+q_B}(1-q_B)^k\gamma > \gamma$$
$$\frac{1-q_G}{1-q_G+q_B}\left(1-q_G^k\right) + \frac{(1-q_G)q_G^k+q_B(1-q_B)^k}{1-q_G+q_B}\gamma > \gamma$$
$$\frac{1-q_G}{q_B} > \left(\frac{1-q_G}{q_B} + \frac{1-(1-q_B)^k}{1-q_G^k}\right)\gamma$$
$$\frac{1-q_G}{q_B} > \frac{1-(1-q_B)^k}{1-q_G^k}\frac{\gamma}{1-\gamma}$$
$$(q_G > 1-q_B\,,\ from\ Condition\ 1)$$
$$\frac{1-q_G}{q_B} > \frac{1-q_G^k}{1-q_G^k}\frac{\gamma}{1-\gamma}$$
$$\frac{1-q_G}{q_B} > \frac{\gamma}{1-\gamma}$$

The last inequality contradicts Condition 1. This proves that for agent $m' = k+1$ it is a best response not to invest when observing $s_{m'} = b$. Note that this result does not depend on k. We can therefore extend it by saying that any agent $m' = m+1 > k$ that observes $s_{m'} = b$ and where all m preceding agents play informatively, best responds by not investing.

Now consider agent $m' = m+1 > k$ which receives signal $s_{m'} = g$ and suppose all m preceding agents play informatively. Let $F_{nX}(l)$ be the cumulative distribution of g signals for n players when the state of nature is X. Then l follows a binomial distribution and in particular F_{nG} is the cumulative distribution of

$B(n, q_G)$, while $F_{n,B}$ is the one for $B(n, 1 - q_B)$.

$$\begin{aligned}
E[u_{m'}(I)|g] &> E[u_{m'}(O)|g] \\
Pr(G|g)\Big[Pr\Big(\big|\{s_j = g,\ j \leq k\}\big| < k \,|\, G\Big) & \\
+\Big(1 - Pr\Big(\big|\{s_j = g,\ j \leq k\}\big| < k \,|\, G\Big)\Big)\gamma\Big] & \\
+Pr(B|g)\Big[\Big(1 - Pr\Big(\big|\{s_j = a,\ j \leq k\}\big| < k \,|\, B\Big)\Big)\gamma\Big] &> \gamma \\
q_G\left[F_{mG}(k-1) + (1 - F_{mG}(k-1))\gamma\right] & \\
+(1-q_B)\left(1 - F_{mB}(k-1)\right)\gamma &> \gamma(q_G + 1 - q_B) \\
q_G(1-\gamma)F_{mG}(k-1) - (1-q_B)\gamma F_{mB}(k-1) &> 0 \qquad (4)
\end{aligned}$$

As long as 4 holds, player m' plays informatively. Next we show that the LHS of 4 is either increasing or quasi-convex with respect to m.

$$\begin{aligned}
E[u_{m'}(I)|g] &\gtreqless E[u_{m'+1}(I)|g] \\
q_G(1-\gamma)F_{m,G}(k-1) & \\
-(1-q_B)\gamma F_{m,B}(k-1) &\gtreqless q_G(1-\gamma)F_{m+1,G}(k-1) \\
& \quad - (1-q_B)\gamma F_{m+1,B}(k-1) \\
q_G(1-\gamma)[F_{m,G}(k-1) - F_{m+1,G}(k-1)] &\gtreqless (1-q_B)\gamma[F_{m,B}(k-1) - F_{m+1,B}(k-1)] \\
\frac{q_G(1-\gamma)}{(1-q_B)\gamma} &\gtreqless \frac{F_{m,B}(k-1) - F_{m+1,B}(k-1)}{F_{m,G}(k-1) - F_{m+1,G}(k-1)} \qquad (5)
\end{aligned}$$

Let $I_x(\alpha, \beta)$ denote the regularized incomplete beta function. Since F_{mG} and F_{mB} are binomial distributions we have:

$$\begin{aligned}
F_{mG} - F_{m+1G} &= I_{1-q_G}(m-k+1, k) - I_{1-q_G}(m-k+2, k) \\
&= I_{1-q_G}(m-k+1, k) - I_{1-q_G}(m-k+1, k) + \frac{q_G^k(1-q_G)^{m-k+1}}{(m-k+1)B(m-k+1, k)} \\
&= \frac{q_G^k(1-q_G)^{m-k+1}}{(m-k+1)B(m-k+1, k)} \qquad (6)
\end{aligned}$$

Here $B(m-k+1, k)$ represents the beta function. Similarly we get:

$$\begin{aligned}
F_{mB} - F_{m+1B} &= I_{q_B}(m-k+1, k) - I_{q_B}(m-k+2, k) \\
&= \frac{(1-q_B)^k q_B^{m-k+1}}{(m-k+1)B(m-k+1, k)} \qquad (7)
\end{aligned}$$

Thus from 5,6 and 7 we obtain:

$$\frac{q_G(1-\gamma)}{(1-q_B)\gamma} \gtrless \frac{(1-q_B)^k q_B^{m-k+1}}{q_G^k(1-q_G)^{m-k+1}}$$
$$\frac{1-\gamma}{\gamma} \gtrless \left(\frac{1-q_B}{q_G}\right)^{k-1}\left(\frac{q_B}{1-q_G}\right)^{m-k+1}$$

When the RHS is smaller then $E[u_{m'}(I)|g] > E[u_{m'+1}(I)|g]$. It is easy to see that the RHS is increasing in m, since $q_B > 1 - q_G$ (from Condition 1). It is easy to see that for $m = k$ which is the smallest possible value for m the RHS can be smaller than the LHS. We have:

$$\frac{1-\gamma}{\gamma} \gtrless \left(\frac{1-q_B}{q_G}\right)^{k-1}\frac{q_B}{1-q_G}$$
$$\frac{1-qG}{q_B} \gtrless \left(\frac{1-q_B}{q_G}\right)^{k-1}\frac{\gamma}{1-\gamma}$$

Which for sufficiently high k gives $LHS > RHS$. Still, as m grows the inequality must eventually switch and remain switched. This shows that $E[u_{m'}(I)|g]$ may be initially decreasing in m and then becomes increasing. This makes it either an increasing or a quasi-concave function of m.

We now show that it can not be that it is increasing and 4 holds. We do so by contradiction. Suppose it is. Then we have:

$$\frac{\gamma}{1-\gamma}\left(\frac{1-q_B}{q_G}\right)^{k-1}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} > 1 \tag{8}$$

and from 4

$$\frac{1-\gamma}{\gamma}\frac{q_G}{1-q_B} > \frac{F_{mB}(k-1)}{F_{mG}(k-1)} \tag{9}$$

But then:

$$\begin{aligned}
\frac{\gamma}{1-\gamma}\left(\frac{1-q_B}{q_G}\right)^{k-1}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} &< \frac{\gamma}{1-\gamma}\frac{1-q_B}{q_G}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} && \{\text{since } q_G > 1-q_b\} \\
&< \frac{F_{mG}(k-1)}{F_{mB}(k-1)}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} && \{\text{from 9}\} \\
&< \frac{\sum_{i=0}^{k-1}\binom{m}{i}q_G^i(1-q_G)^{m-i}}{\sum_{i=0}^{k-1}\binom{m}{i}(1-q_B)^i q_B^{m-i}}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} \\
&< \frac{\sum_{i=0}^{k-1}\binom{m}{i}q_G^i(1-q_G)^{k-1-i}}{\sum_{i=0}^{k-1}\binom{m}{i}(1-q_B)^i q_B^{k-1-i}} \\
&= 1
\end{aligned}$$

Which contradicts 8! This shows that when $E[u_{m'}(I)|g]$ is increasing, 4 does not hold. Since we already showed that $E[u_{m'}(I)|g]$ becomes increasing in m as m grows, this shows that eventually as it does so an agent will not play informatively and so will all agents after him. Notice that the LHS of 4 goes to zero as m grows. This means the inequality never switches back. After one agent switches away from informative play, so do all agents after him.

□

Proof. (**Proposition 2)** Let $W(m)$ denote the sum of expected utilities from a pure strategy profile in which m agents play informatively. For the equilibrium profile we have:

$$W\left(\hat{m}(k,q_A,q_B)\right) = \sum_{m=1}^{\hat{m}(k,q_A,q_B)} E[u_m(x)] + (N-\hat{m}(k,q_A,q_B))\gamma$$

From the proof of proposition one we know that for an agent m playing informatively: $E[u_m(x)] > \gamma$ for $m < \hat{m}(k,q_A,q_B)$ and $E[u_m(x)] < \gamma$ for $m < -\hat{m}(k,q_A,q_B)$. It is therefore immediate to see that:

$$\begin{aligned}
\sum_{m=1}^{m'} E[u_m(x)] + (N-m')\gamma &< \sum_{m=1}^{\hat{m}(k,q_A,q_B)} E[u_m(x)] + (N-\hat{m}(k,q_A,q_B))\gamma \\
&< \sum_{m=1}^{m''} E[u_m(x)] + (N-m'')\gamma
\end{aligned}$$

Which summarizes to:

$$W(m') < W\left(\hat{m}(k, q_A, q_B)\right) < W(m'')$$

for $m' < \hat{m}(k, q_A, q_B) < m''$. Also note that this is independent of N, which proves the proposition. □

Proof. (**Proposition 3)**

$$\begin{aligned} E_{k+1}[u_m(A)|a] &> E_k[u_m(A)|a] \\ q_A F_{mA}(k+1) + (1-q_B)(1-F_{mB}(k+1)) &> q_A F_{mA}(k) + (1-q_B)(1-F_{mB}(k)) \\ \frac{q_A}{1-q_B} &> \frac{F_{mB}(k+1) - F_{mB}(k)}{F_{mA}(k+1) - F_{mA}(k)} \\ &> \frac{f_{mB}(k+1)}{f_{mA}(k+1)} \\ &> \frac{\binom{m}{k+1}(1-q_B)^{k+1} q_b^{m-k-1}}{\binom{m}{k+1} q_A^{k+1}(1-q_A)^{m-k-1}} \\ 1 &> \left(\frac{1-q_B}{q_A}\right)^k \left(\frac{q_B}{1-q_A}\right)^{m-k} \end{aligned} \tag{10}$$

From the proof of proposition 1 we know that 10 must hold for $\hat{m}$. This player is the first (in order of priority) that herds. We show that increasing k increases $\hat{m}$'s expected payoff from playing informatively and therefore he eventually switches to that strategy. This makes $\hat{m}+1$ the first player to herd. □

Proof. (**Proposition 4)** We know from proposition 1 that given Condition 1 any agent that receives signal $s_i = b$ best replies by not investing. Let σ denote the probability with which an agent decides to play informatively. For an agent that receives signal $s_i = g$ the expected payoff from investing given that all other agents play strategy $\sigma \in [0,1]$ is:

$$\begin{aligned} E_\sigma[u(I)|g] = &\frac{q_G}{q_G + 1 - q_B}\left[\frac{k}{N} + \frac{1}{N}\sum_{m=k+1}^{n-1}\sum_{i=0}^{k-1}\binom{m}{i}\sigma^i(1-\sigma)^{m-i}\right] \\ &+ \frac{1}{N}\sum_{m=k+1}^{n-1}\sum_{i=k}^{m}\binom{m}{i}\sigma^i(1-\sigma)^{m-i}E[u_{m+1}(I)|g] \end{aligned}$$

It is easy to sea that given the properties of the binomial distribution and the fact that as was shown in the proof of proposition 1 $E[u_{m+1}(I)|g] \leq \frac{q_G}{q_G+1-q_B}$, the above expression is decreasing in σ. The symmetric equilibrium strategy σ^* is the one that solves the following equation:

$$E_{\sigma^*}[u(I)|g] = \gamma \tag{11}$$

Note that for $\sigma = 0$ we have:

$$E_{\sigma}[u(I)|g] = \frac{q_G}{q_G + 1 - q_B} > \gamma$$

And for $\sigma = 1$ we obtain:

$$E_{\sigma}[u(I)|g] = \frac{q_G}{q_G + 1 - q_B}\frac{k}{N} + \frac{1}{N}\sum_{m=k+1}^{n-1} E[u_{m+1}(I)|g]$$

The RHS in the last expression can be less than γ if k is low enough or γ is high enough. When this is the case, 11 has a unique solution $\sigma^* \in (0, 1)$. Otherwise, the unique symmetric equilibrium obtains for $\sigma^* = 1$. □

Proof. (**Proposition 5)** Consider a strategy profile in which $\tilde{m}$ agents play informatively and all others herd. It is easy to see that $\tilde{m} < k$ can not be an equilibrium profile. Suppose it were. Then an agent i that herds and observes $s_i = g$ is better of investing since he can obtain a slot and his payoff will be $E[u_i(I)|g] = \frac{q_G}{q_G+1-q_B} > \gamma = E[u_i(O)|g]$. Thus it is not an equilibrium.

Consider $\tilde{m} \geq k$. Suppose i is in the set of agents that play informatively and receives signal $s_i = g$. His expected payoff from investing is:

$$E_{\tilde{m}}[u(I)|g] = \frac{q_G}{q_G + 1 - q_B}\frac{k}{\tilde{m}} + \frac{1}{\tilde{m}}\sum_{m=k+1}^{\tilde{m}-1} E[u_m(I)|g] \tag{12}$$

For $\tilde{m}$ to characterize an equilibrium profile it must be that:

$$E_{\tilde{m}}[u(I)|g] \geq \gamma$$
$$E_{\tilde{m}+1}[u(I)|g] < \gamma$$

The first of these two conditions guarantees that no agent playing informatively has an incentive to deviate. The second does the same for the agents herding. We know that these conditions are necessary and sufficient from the properties of $E[u_m(I)|g]$ derived in the proof of proposition 1. It is immediate to see that $\tilde{m} < \hat{m}(k, q_A, q_B) - 1$ cannot be an equilibrium. By definition, $E[u_i(I)|g] > \gamma$, $\forall\ i < \hat{m}(k, q_A, q_B)$. Thus, such a profile would violate the second of the equilibrium conditions above. □

Proof. (**Proposition 6**) Let:

$$g_A(m,k) = \begin{cases} 1\ , & m \leq k \\ F_{mA}(k)\ , & m > k \end{cases}$$

and

$$g_B(m,k) = \begin{cases} 1\ , & m \leq k \\ F_{mB}(k)\ , & m > k \end{cases}$$

Then, given k:

$$\begin{aligned} &E_t[u_{m+1}(A)|a] = \\ &= \frac{q_A}{q_A + 1 - q_B}\left[\sum_{m=0}^{t}\binom{t}{m}p^m(1-p)^{t-m}\cdot g_A(m,k)\right] \\ &+ \frac{1-q_B}{q_A+1-q_B}\left[1 - \sum_{m=0}^{t}\binom{t}{m}p^m(1-p)^{t-m}\cdot g_B(m,k)\right] \\ &= \frac{1}{q_A+1-q_B}\left[\sum_{m=0}^{t+1}\binom{t}{m}p^m(1-p)^{t-m}\left(q_A g_A - (1-q_B)g_B\right) + (1-q_B)\right] \\ &= \frac{1}{q_A+1-q_B}\left[\sum_{m=0}^{t+1}\binom{t}{m}p^m(1-p)^{t-m}E[u_{m+1}(A)|a] + (1-q_B)\right] \end{aligned}$$

Notice that for $m < \hat{m}(k, q_A, q_B)$, $E_t[u_{m+1}(A|a] > 0$, thus the above expression is positive. This means that all agents arriving at $t < \hat{m}(k, q_A, q_B)$ play informatively. This proves the minimum bound on $\hat{m}(k, q_A, q_B)$. From now on consider $t \geq \hat{m}(k, q_A, q_B)$. Remember that $E[u_{m+1}(A)|a]$ is a quasi-concave function. Let $\tilde{m}$ be such that this function is decreasing for $m \leq \tilde{m}$ and increasing for $m > \tilde{m}$. From the proof of proposition 1 we have that $\tilde{m} > \hat{m}$. Then we have:

$$\begin{aligned} &(q_A + 1 - q_B)E_t[u_{m+1}(A)|a] = \\ &+ \sum_{m=0}^{\tilde{m}}\binom{t}{m}p^m(1-p)^{t-m}E[u_{m+1}(B)|a] + \sum_{m=\tilde{m}}^{t}\binom{t}{m}p^m(1-p)^{t-m}E[u_{m+1}(B)|a] + (1-q_B) \end{aligned}$$

From stochastic dominance for the Bernoulli distribution we have that the first summation is decreasing in t. The second summation is always negative. The last term is constant with respect to t. Thus, as t increases, the expected payoff crosses zero at most once. $\hat{t}(k, q_A, q_B, p)$ is such that the expected payoff is negative for this value and positive for all smaller t. The agent arriving at $\hat{t}(k, q_A, q_B, p)$ ignores his private information and herds. All agents arriving after $\hat{t}(k, q_A, q_B, p)$ have the same expected payoff and herd as well. $\square$