Graduate Program in Economics

IDEA

Course: **Probability and Statistics**

Faculty: **Jordi Caballé**

Term: First Semester

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Office Hours: Upon request. Send an e-mail for an appointment.

Description:

This course will cover the topics of probability theory, random variables, distributions, and stochastic processes using measure theory. It will also cover the traditional topics of sampling, estimation, and hypothesis testing, which are needed for undertaking empirical research.

Slides and exercises can be downloaded from the professor's webpage

Objective:

The objective of this course is to provide the mathematical foundations and the tools of statistics that will be used in courses dealing with Economics of Uncertainty and Econometrics.

Outline:

1. Probability

Combinatorics. Events and measurable spaces. Probability. Conditional probability. Theorem of total probability. Bayes' theorem.

2. Measure Theory

Lebesgue measure. Lebesgue-Stieltjes measures and distribution functions. Measurable functions. Integral with respect to a measure. Absolute continuity of measures and the Radon-Nikodym theorem. Product measures and Fubini's theorem.

3. Random Variables and Distributions

Random variables. Probability distributions. Distribution function of a random variable. Discrete random variables and probability functions. Absolutely continuous random variables and densities. Random vectors and marginal distributions. Independent random variables. Generalized conditional probability and distribution.

4. Expectation

Mathematical expectation. Moments. Chebyshev's inequality. The moment-generating function of a random variable. Product moments. Mean and variance of linear combinations of random variables. Conditional expectation. The law of iterated expectations. Jensen's inequality.

5. Special Distributions

The discrete uniform, Bernoulli, binomial, Pascal, geometric, and hypergeometric distributions. The multinomial and multivariate hypergeometric distribution. Integration by parts and by change of variable to polar coordinates. The uniform, gamma, exponential, chi-square, and beta distributions. The normal and the multivariate normal distributions.

6. Functions of Random Variables

The distribution of a function of a random variable. The probability function of a function of a random variable. The density of a function of a random variable. Characteristic functions, moment-generating functions, and Laplace transforms of functions of random variables.

7. Stochastic Processes and Limiting Distributions

Stochastic processes. Filtrations and martingales. Markov processes. Convergence in probability, in mean square, in distribution, and almost sure convergence. Convergence of distribution functions and of probability measures. The Poisson distribution as the limit of binomial distributions. The standard normal distribution as the limit of standardized binomial distributions. Laws of large numbers. The central limit theorem.

8. Sampling

Random samples and statistics. The distribution of the sample mean. The distribution of the variance of a random sample from a normal population. The *t* distribution. The *F* distribution.

9. Estimation

Point estimation. Sample mean and sample variance. The Cramér-Rao lower bound for unbiased estimators. Asymptotic properties of estimators: consistent estimators. Sufficient estimators. The method of moments. Maximum likelihood estimation. Bayesian estimation. Interval estimation.

10. Hypothesis Testing

Statistical hypotheses and their tests. The power function of a test. The Neyman-Pearson lemma. Likelihood ratio tests. Acceptance intervals. The *p*-value of a statistical test. Contingency tables. Goodness of fit.

References:

Ash, R.B., *Real Analysis and Probability,* Academic Press.

Bierens, H.J*., Introduction to the Mathematical and Statistical Foundations of Econometrics*, Cambridge University Press.

Billingsley, P., *Probability and Measure,* Wiley.

DeGroot, M.H. and Schervish, M.J. *Probability and Statistics,* Pearson.

Hogg, R.V., McKean, J. and Craig, A.T., *Introduction to Mathematical Statistics,* Pearson.

Lindgren, B.V., *Statistical Theory,* Chapman and Hall/CRC.

Grading:

Students must solve a series of problem sets. Problem sets will have a weight of 20% in the final grade. There will be a final exam, which will have a weight of 80% in the final grade.

Note: Some of this year's exercises have appeared on this course previously. Thus, it is possible, even likely, that you might be able to obtain solutions to these exercises that I have handed out earlier. However, I strongly recommend you not to look at these solutions when solving the exercises. By handing in your answers, you declare that the solutions are your own and that they are not based on solutions from previous years. If I catch you cheating, I will give you 0 points from the exercises. Even more significantly, you will suffer a reputation loss within IDEA (and academia in general) by presenting someone else's work as your own. Check out the definition of "plagiarism" and how it is viewed in academic circles if you do not immediately grasp what the consequences of cheating will be.