

**What are the effects of fiscal shocks?**  
**III: Models with labor market frictions**

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## References

Bruckner, M. and Pappa, P. 2010, Fiscal expansions affect unemployment, but they may increase it, CEPR working paper, 7766.

Farmer, R. , 2009, Fiscal policy can reduce unemployment: but there are less costly and more effective alternatives, NBER working paper 15931

Monacelli, T. , Perotti, R. and Trigari, A., 2010, Unemployment fiscal multipliers, NBER working paper 15931

Andolfatto, D., 1996, Business cycles and labor market search, American Economic Review, 86, 112-132.

Hall, R., 2005, Job loss, Job finding and unemployment in the US economy over the past 50 years, in M. Gerlter and K. Rogoff (eds.) NBER Macroeconomic Annual, MIT Press, 101-137.

Merz, M., 1995, Search in the labor market and real business cycles, *Journal of Monetary Economics*, 36, 269-300

Shimer, R. ,2005, The cyclical behavior of equilibrium unemployment and vacancies, *American Economic Review*, 95, 25-49.

Shimer, R.,2009, *Labor markets and the business cycle*, Princeton University Press

Trigari, A., 2009 Equilibrium unemployment, job flows and inflation dynamics, *Journal of Money, Credit and Banking*, 41, 1-33.

Walsh, C., 2005, Labor market search, sticky prices and interest rate rules, *Review of Economic Dynamics*, 8, 829-849.

## Outline

- Matching frictions in a RBC model.
- Matching frictions in a NK model.
- Adding labor market participation decisions.

# 1 Introduction

- Models so far considered have nothing to say about unemployment and hiring/firing - labor market is always in equilibrium. They allow, at most, to evaluate how fiscal policy decisions affect output and its components.
- Most of current fiscal programs are especially designed to target job creation (unemployment reductions).
- What kind of models can help us to understand these issues? Mortensen-Pissarides matching friction model is now the workhorse. Are there other models which could be used?
- Is it possible that fiscal expansions increase unemployment? Under what conditions?

## 2 A RBC model with search frictions

- This model has been used to interpret the dynamics following technology shocks starting with Merz (1995) and Andolfatto (1996) and, recently, by Shimer (2005) and Hall (2005).
- Some attempts also to use this framework to analyze monetary policy shocks, see Walsh (2005) or the dynamics of inflation see Trigari (2009).
- Natural to ask what this model would tell us about the effects of fiscal shocks on unemployment, vacancies and job finding rates.
- Some doubts about the empirical relevance of labor market friction models, but intense area of research.

## 2.1 The labor market structure

- Continuum of infinitely-lived agents and infinitely-lived firms, both of measure one.
- Firms employ  $n_t$  workers and post  $v_t$  vacancies. There is a constant cost  $\kappa$  per posted vacancies.
- Unemployment is  $u_t = 1 - n_{t-1}$ .
- New hires are produced with  $m_t = \gamma_m u_t^\gamma v_t^{1-\gamma}$ , where  $\gamma_m$  measures the efficiency of the matching process (for  $\gamma_m \rightarrow 1$  more efficient process).

- Probability of filling a vacancy is  $q_t = m_t/v_t = \gamma_m \theta_t^{-\gamma}$ , where  $\theta_t = \frac{v_t}{u_t}$  measures labor market tightness (ratio of vacancies to unemployment).
- Probability of finding a job is  $p_t = m_t/u_t = \gamma_m \theta_t^{1-\gamma}$ .
- $p_t, q_t$  are exogenous to agents/firms decisions.
- An exogenous fraction  $1 - \rho$  of the existing matches separates at each  $t$  (thus  $\rho$  is the survival probability of a worker with a firm).

## Firms

- Output production:  $y_t = z_t k_t^\alpha n_t^{1-\alpha}$ . The capital market is competitive.
- Timing: at each  $t$  start with  $n_{t-1}$  employed, post  $v_t$  vacancies, when there are  $u_t$  unemployed. There are  $m_t$  new hires and  $(1 - \rho)$  workers are exogenously separated and become unemployed. The new matches become productive within the period and can not get separated until the next period. Thus, the available work force is

$$n_t = \rho n_{t-1} + q_t v_t \quad (1)$$

- Firm problem

$$F(n_{t-1}, k_t) = \max_{k_t, n_t} (y_t - w_t n_t - \kappa v_t - r_t k_t + \beta E_t \Lambda_{t,t+1} F(n_t, k_{t+1})) \quad (2)$$

subject to (1) where  $\beta \Lambda_{t,t+1}$  is the stochastic discount factor (to be defined later).

- Problem is dynamic: current hires give future value to the firm.

Optimality:

$$\begin{aligned}\frac{\kappa}{q_t} &= a_t - w_t + \rho\beta E_t(\Lambda_{t,t+1} \frac{\kappa}{q_{t+1}}) \\ F_{n,t} &= a_t - w_t + \rho\beta E_t(\Lambda_{t,t+1} F_{n,t+1})\end{aligned}\quad (3)$$

where  $a_t = (1 - \alpha)\frac{y_t}{n_t}$ . The second expression comes from the fact that  $F_{n,t} = \frac{\kappa}{q_t}$  (the derivative of  $F$  with respect to  $n_t$ ) is the value to the firm of having an additional worker at  $t$  after new workers have joined (i.e. after the vacancy costs are paid).

- Marginal cost of hiring a worker = marginal benefit (expected discounted net earnings for this worker)

## Households

- Pool family income and allocate consumption to maximize the sum of utility of members.

- Employed utility:  $\frac{[c_{e,t}^{1-\sigma}(1+(\sigma-1)b)^\sigma - 1]}{1-\sigma}$ : unemployed utility:  $\frac{c_{u,t}^{1-\sigma} - 1}{1-\sigma}$ , where  $b$  is the relative disutility of work,  $\sigma > 0$  controls substitutability of consumption and leisure (for  $\sigma = 1$  separable utility).

- Capital accumulation  $k_{t+1} = (1-\delta)k_t + i_t(1-\phi_t)$ , where  $\phi_t \equiv \phi(\frac{i_t}{i_{t-1}} - 1)$  and  $\phi(1) = \phi'(1) = 0$  and  $\phi''(1) > 0$ .

- Budget constraint

$$c_t + i_t + E_t(\Lambda_{t,t+1}B_{t,t+1}) \leq w_t n_t + r_t k_t + B_t + \pi_t - T_t \quad (4)$$

where  $c_t = c_{e,t}n_t + c_{u,t}(1 - n_t)$ ,  $B_{t,t+1}$  is the holding of real one period state contingent securities and  $\pi$  are profits from the firms and  $\tau_t$  lump sum taxes.

- Household recognize that employment flow of its member is regulated by

$$n_t = \rho n_{t-1} + p_t(1 - n_{t-1}) \quad (5)$$

- Shimer (2009): setup is equivalent to one where there is a representative household with utility

$$E_t \sum_t \beta^t \frac{[c_t^{1-\sigma}(1 + (\sigma - 1)bn_t)^\sigma - 1]}{1 - \sigma} \quad (6)$$

- Household problem: maximize (6) subject to (4) and (5). Optimization problem defines a value function  $H(n_{t-1}, k_t)$ .

- Optimality conditions

$$\lambda_t = \left( \frac{1 + (\sigma - 1)bn_t}{c_t} \right)^\sigma \quad (7)$$

$$\psi_t \left( 1 - \phi_t - \frac{i_t}{i_{t-1}} \phi_{i,t} \right) = 1 - \beta E_t (\psi_{t+1} \Lambda_{t,t+1} \left( \frac{i_{t+1}}{i_t} \right)^2 \phi_{i,t+1}) \quad (8)$$

$$\psi_t = \beta E_t \Lambda_{t,t+1} (r_{t+1} + \psi_{t+1} (1 - \delta)) \quad (9)$$

$$H_{n,t} = \lambda_t w_t - U_{n,t} + \beta (\rho - p_{t+1}) E_t H_{n,t+1} \quad (10)$$

where  $\phi_{i,t} = \frac{\partial \phi_t}{\partial i_t}$ . First equation defines the MU of wealth; the second is the optimality condition for investment ( $\psi_t$  is the multiplier on the capital accumulation equation); the third the optimality condition for capital.

-  $H_{n,t}$  is the marginal value of having one member employed rather than unemployed and  $U_{n,t} = \sigma b \left( \frac{1 + (\sigma - 1)bn_t}{c_t} \right)^{\sigma - 1}$ . Value of having an additional employed worker has three terms:

i) increased in utility, given by the real wage,

ii) decrease in utility due to lower leisure,

iii) the continuation value (utility contribution of continuing the match).

Optimal employment decision can also be written as:

$$H_{n,t} = \lambda_t(w_t - \omega_t) + \beta(\rho - p_{t+1})E_t H_{n,t+1} \quad (11)$$

where  $\omega_t = \frac{U_{n,t}}{\lambda_t}$  is the current marginal value of non-work activities (leisure plus home production and unemployment benefits, etc.).

- Elasticity of  $\omega_t$  with respect to  $\lambda_t$  (MU of wealth) is decreasing in  $\sigma$  - the parameter regulating the complementarity between consumption and leisure. Keep this for later when we change  $\sigma$  from baseline.

## Bargaining over the surplus and reservation wage

- Wage chosen to  $\max H_{n,t}^\vartheta F_{n,t}^{1-\vartheta}$ , where  $\vartheta \in (0, 1]$  reflects bargaining power. Optimality  $\vartheta F_{n,t} = (1 - \vartheta) \frac{H_{n,t}}{\lambda_t}$ .

$-\frac{H_{n,t}}{\lambda_t}$  can also be interpreted as the marginal benefit for the household to have one additional worker, expressed in consumption units. Each agent gets a constant share of the surplus  $S_t = F_{n,t} + \frac{H_{n,t}}{\lambda_t}$ ; the size depend on  $\vartheta$ .

- Size of the surplus related to the bargaining set (gap between reservation wage of workers and maximum acceptable wage by firm), i.e.

$$S_t = w_t^f - w_t^h \quad (12)$$

$$w_t^f = a_t + \rho\beta E_t(\Lambda_{t,t+1} F_{n,t+1}) \quad (13)$$

$$w_t^h = \omega_t - \beta E_t((\rho - p_{t+1})\Lambda_{t,t+1} H_{n,t+1}) \quad (14)$$

- Household reservation wage decrease in the continuation value (willing to take lower wages if they see prospects for future). Firm maximum wage increase in the continuation value

- Bargained wage:

$$w_t = \vartheta w_t^f + (1 - \eta)w_t^h \quad (15)$$

- Another expression of the surplus:

$$S_{n,t} = (a_t - \omega_t) + \beta E_t((\rho - \vartheta p_{t+1})\Lambda_{t,t+1}S_{n,t+1}) \quad (16)$$

Two terms: current gap between MP of labor and value of non-work activities; future surplus of the match net of worker's future surplus if the match breaks.

- Optimal hiring condition:

$$\frac{\kappa\theta_t^\gamma}{\gamma_m} = (1 - \vartheta)S_{n,t} \quad (17)$$

$$= (1 - \vartheta)(a_t - \omega_t) + \beta E_t((\rho - \vartheta p_{t+1})\Lambda_{t,t+1} \frac{\kappa\theta_{t+1}^\gamma}{\gamma_m}) \quad (18)$$

The second equation expresses hiring decisions in terms of labor market tightness.

- For  $\kappa \rightarrow 0$  (no vacancy costs) or  $\gamma_m \rightarrow \infty$  (perfect efficiency in matching) condition is equivalent to MP of labor = MU of leisure. Thus a frictionless RBC model is a special case of this model with labor market frictions.

## 2.2 Government

$$T_t = g_t \quad (19)$$

$$\log g_t = (1 - \rho_g) \log \bar{g} + \rho_g \log g_{t-1} + \epsilon_{g,t} \quad (20)$$

$\bar{g}$  is the steady state expenditure to output ratio.

Resource constraints

$$y_t = c_t + g_t + i_t + \kappa v_t \quad (21)$$

## 2.3 Some Intuition

- A log linear approximation of (18) gives

$$\hat{\theta}_t = \frac{1 - \zeta}{\gamma} \left( \frac{\hat{a}_t}{1 - \bar{w}} - \frac{\bar{w}\hat{\omega}_t}{1 - \bar{w}} \right) - \frac{\zeta\hat{r}_t}{\gamma} + \beta \frac{\rho\gamma - \vartheta p}{\gamma} E_t \hat{\theta}_{t+1} \quad (22)$$

where  $\zeta = \beta(\rho - \vartheta p) \Rightarrow 0$ ,  $\hat{r}_t = -E_t \hat{\Lambda}_{t,t+1}$ ,  $\bar{w} = \frac{\omega}{a} = \frac{\sigma b \lambda^{-1/\sigma}}{a}$  and hats denote percentage deviations from steady states.

- $\bar{w}$  increasing in  $b$  (disutility of work), decreasing in  $\lambda$  (MU of wealth).
- Variations on government spending affect the surplus and thus the hiring rate via three channels:
  - a) Marginal value of work activities.

If  $G$  increases, household budget constraint is tightened and  $\lambda_t$  increases and this lower the value of non-working activities  $\omega_t$  (as in the frictionless RBC model), increases the surplus,  $F_{n,t}$  and the hiring rate  $\theta_t$ . The size of this effect depend on the size of  $\bar{\omega}$  and  $\sigma^{-1}$  (comparable to the elasticity of labor supply in standard RBC).

b) real interest rate

If  $G$  increases and  $\lambda_t$  increases, the equilibrium interest rate increases and this produces a fall in the discounted marginal benefit of a new hire. This reduces the surplus and discourages hiring.

c) capital accumulation.

If  $G$  increase and  $\lambda_t, r_t$  increase, investment fall implying a future fall in the marginal product of labor and this discourages hiring.

d) direct effect -crowding out of vacancies

- b)-c), d) are absent in a standard RBC.
- Would G spending reduce unemployment and stimulate employment? It depends on whether a) is larger than b)+c). Shimer (2005): for this to happen need  $\bar{w}$  to be large.
- A large  $\bar{w}$  reduces the surplus and produces larger labor market fluctuations in response to shocks (i.e., with large  $\bar{w}$  employment is elastic to shocks).  $\bar{w}$  similar to Frish elasticity in RBC models.

## 2.4 Dynamics

Parameters (monthly matching)

$\beta = 0.99^{0.33}$ ;  $\delta = 0.025/3$ ;  $\alpha = 0.33$ ,  $\sigma = 1$  (separable utility).

$\eta_k = 3.24$  (investment adjustment cost parameter);  $\rho = 0.9$  (survival rate of the match);  $\gamma = 0.4$  (elasticities of matches to unemployment),  $\vartheta = 0.5$  (bargaining power);

$\rho_g = 0.9^{0.33} \gamma_m$  (efficiency of matching) set to target  $\theta = 0.5$ ;  $\kappa$  (vacancy cost) set to target  $p = 0.45$ ;  $b$  (unemployment flow value) set to target  $\bar{\omega} = 0.9$  (high side of estimates).

- If  $b$  is set to target lower values of  $\bar{\omega} = 0.4, 0.75$ , effects are muffled.

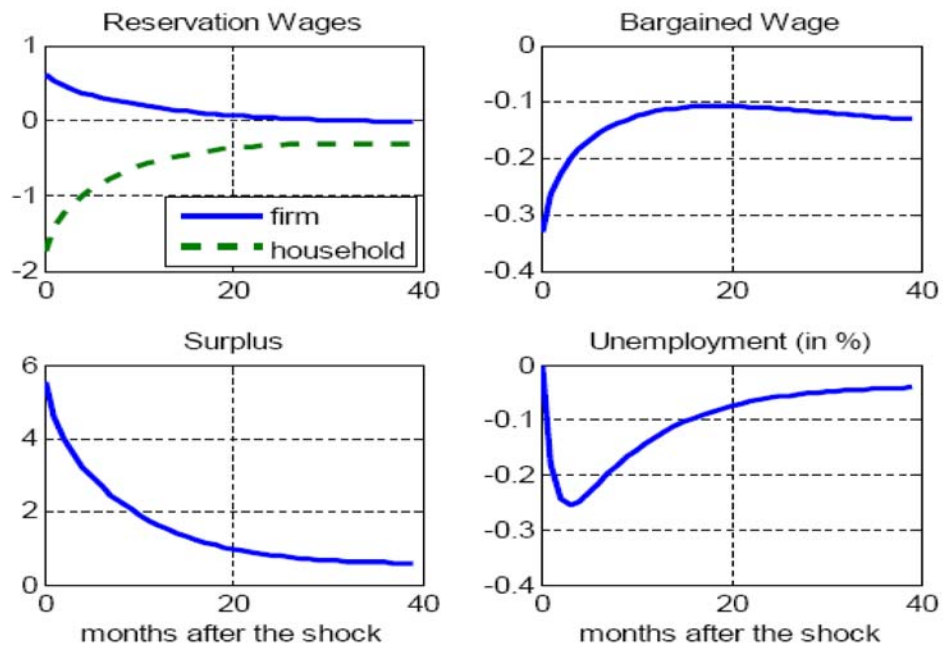


Figure 3: Responses to a rise of government spending equal to 1% of GDP.

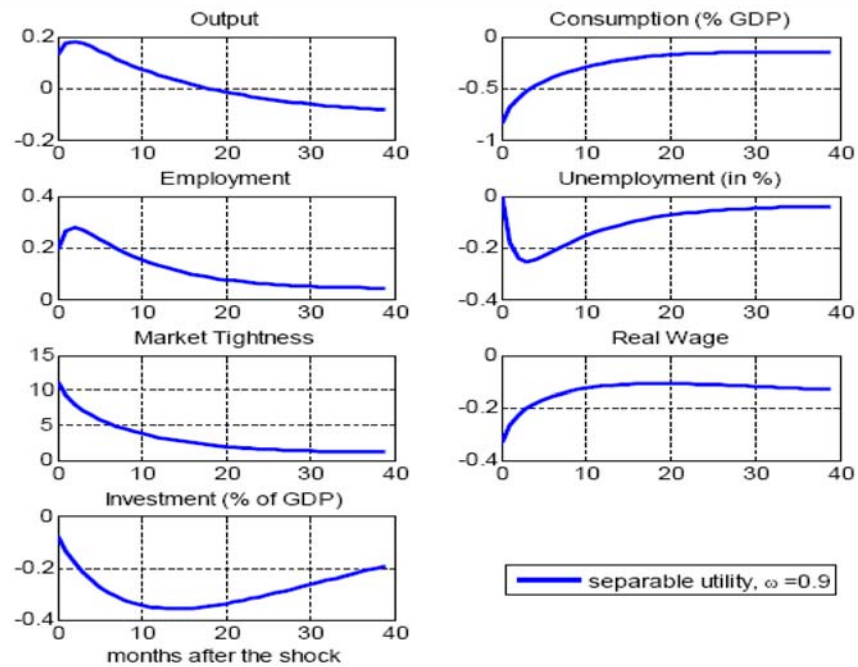


Figure 4: Responses to a rise of government spending equal to 1% of GDP.

- Surplus increases (household reservation wage falls, firms maximum wage increases)
- Firms maximum wage increases because even if MP labor falls, the value of continuation increases
- Household reservation wage falls because  $\omega_t$  falls and expected continuation increases. This induces a fall in the bargaining wage.
- Increase in labor market tightness and fall of unemployment; output and employment up, consumption and real wages down.
- Apart from unemployment/hiring, dynamics similar to those of an RBC model.

## 2.5 Extensions

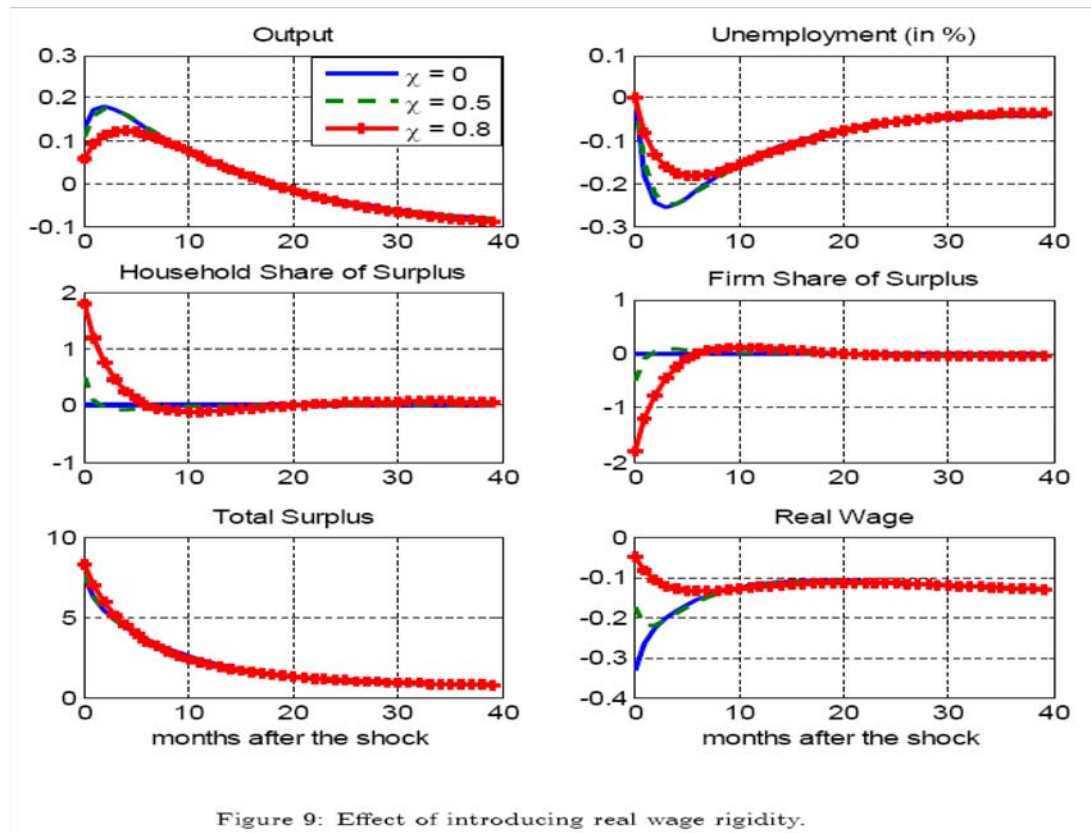
1) Non-separability: complementarities between consumption and leisure dampens the responses (the marginal value of non-work activities is less sensitive to the marginal utility of wealth).

2) Unemployment benefits. So far  $\omega$  reflects only the value of leisure. If unemployment benefits are added, channel a) is much smaller (or even inexistent) i.e. increases in  $G$  lead to a fall in hiring, employment, etc.

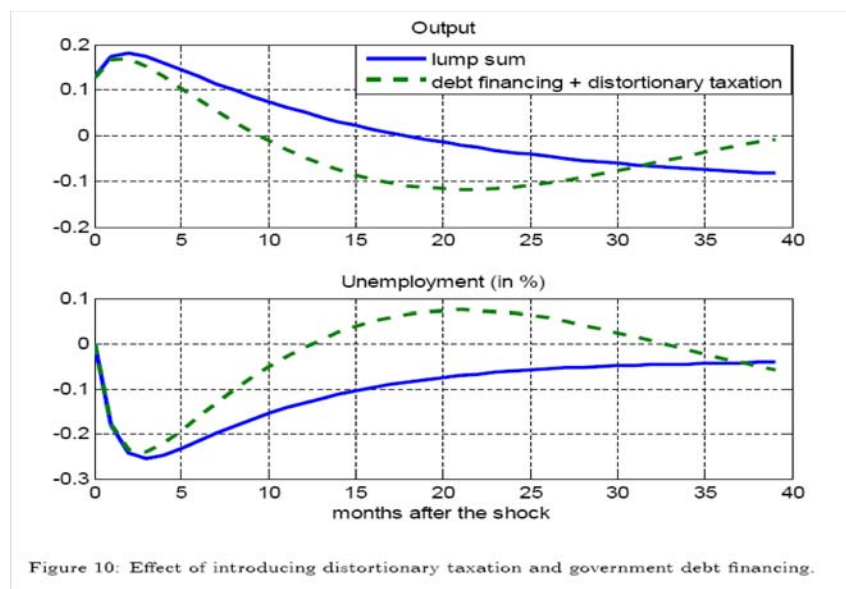
3) Wage rigidities. So far use Nash bargaining to split surplus. Could use any other assumption, e.g. rigid wages, to decide the split.

Rigid wages do not affect match after firms and workers are matched. But they affect the rate at which firms post vacancies (they change the gains from hiring a worker). If wages are perfectly fixed, surplus of a firm decreases after an increase in  $G$ , discouraging hiring (dampening effect).

Let  $w_t = (1 - \chi)w_t^{wb} + \chi w_{t-1}$ .  $\chi$  partial adjustment parameter ( $\chi = 0$  no rigidities).



4) Distorting labor taxation: it reduces surplus (by the amount  $\tau_{nt}w_t$ ).  
Two effects: i) It increases the value of non-work activities and discourage hiring.; ii) it changes the future relative bargaining power.



### 3 A NK model with search frictions

- In a NK key to understand dynamics is the behavior of markups.
- Any shock that boost output and marginal costs makes markups countercyclical when there is price stickiness. This shifts the MP of labor curve and reinforces the employment effects stemming on the wealth effect of labor supply (i.e. both labor demand and labor supply move).
- What is the effect of labor market frictions in a NK model? Use the Walsh-Trigari setup.

- Assume there are search frictions in the intermediate good sector.
- Add price rigidities in a monopolistic retail sector. Retailers buy intermediate goods in competitive markets, differentiate them and sell them to the public.
- Retailers adjust prices using a standard Calvo lottery.
- Monetary authority has a Taylor rule

$$(1 + R_t) = (1 + R)\pi_t^{\iota_1} \left(\frac{y_t}{y_{t-1}}\right)^{\iota_2} \quad (23)$$

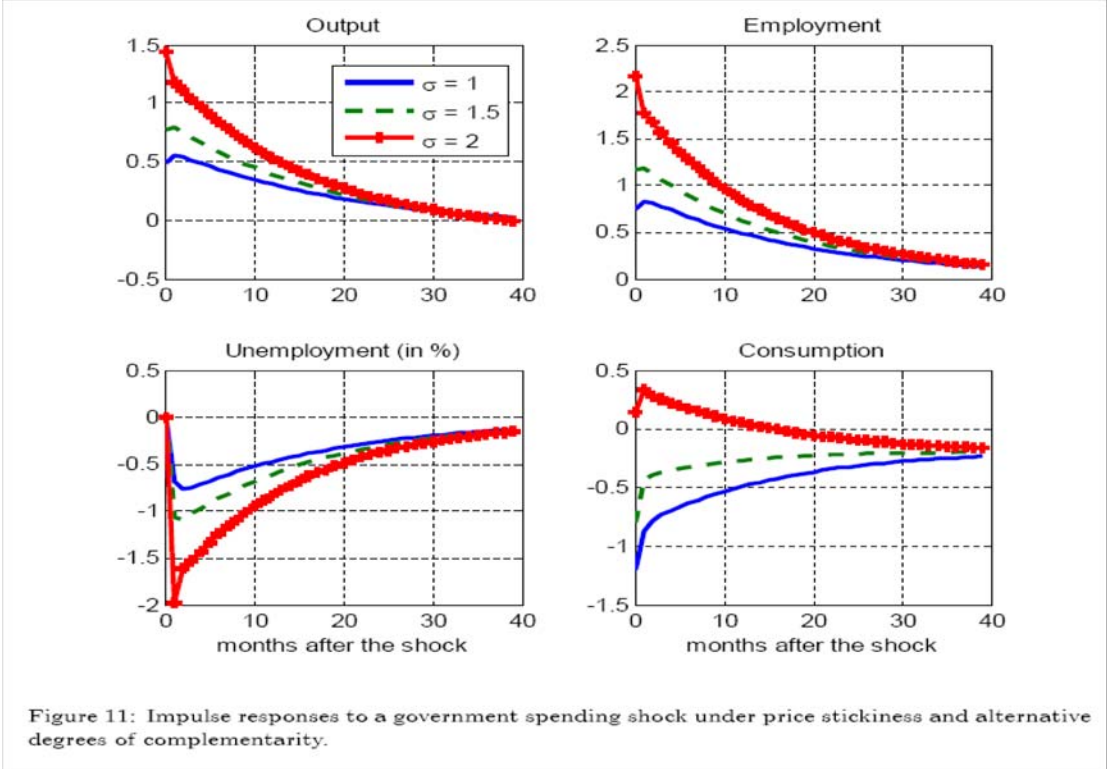
where  $\pi_t$  is final consumption good inflation,  $\iota_1 > 1, \iota_2 > 0$ .

- MP of labor in intermediate good sector (expressed in terms of final goods) is

$$a_t^* = \mu_t^{-1} a_t \quad (24)$$

where  $\mu_t$  is the price markup. Thus, changes in the markup will affect the surplus because the MP of labor changes ( recall the surplus is  $S_{n,t} = (a_t^* - \omega_t) + \beta E_t((\rho - \vartheta p_{t+1}) \Lambda_{t,t+1} S_{n,t+1})$ ).

- Countercyclical movements in the markup (induced by any shocks, not only G shocks) increase the effective MP of labor. Since hiring depends on current and expected future MP of labor, G shocks will boost hiring and employment.



- Effect of complementarity goes in the opposite direction as in a RBC model. Output, unemployment and employment responses larger with complementarities between consumption and labor.
- Even without complementarities, effects are larger than in a RBC model.
- Consumption increases when  $\sigma = 2$ . Intuition (more in the next set of lectures) two effects: if prices are rigid and monetary policy responds to inflation (so as to keep the real interest rate constant), consumption must be constant. But as employment increases, complementarity between consumption and employment makes also consumption increase.
- In this model consumption is responsive to employment not to interest rates (which are constant). Big change relative to a standard growth model.

## **4 Labor market participation - insiders/outside**

- In the previous models labor market participation is not considered.
- Does labor market participation fluctuate over business cycles? Yes.
- Does labor market participation respond to shocks in a way that may change conclusions obtained with fixed labor market participation? Maybe.
- Are labor market non-participants different? In what way?

- What happens in labor markets after a government spending shock?

Figure 1. The Effect of Government Expenditure Shocks on the Unemployment Rate (Baseline Estimates)

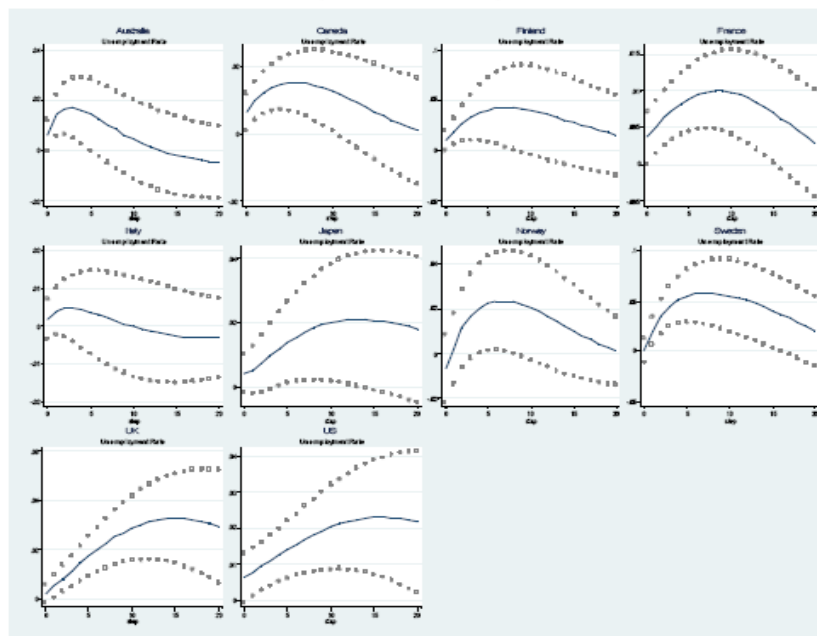


Figure 7. The Effect of Government Expenditure Shocks on Real Wages

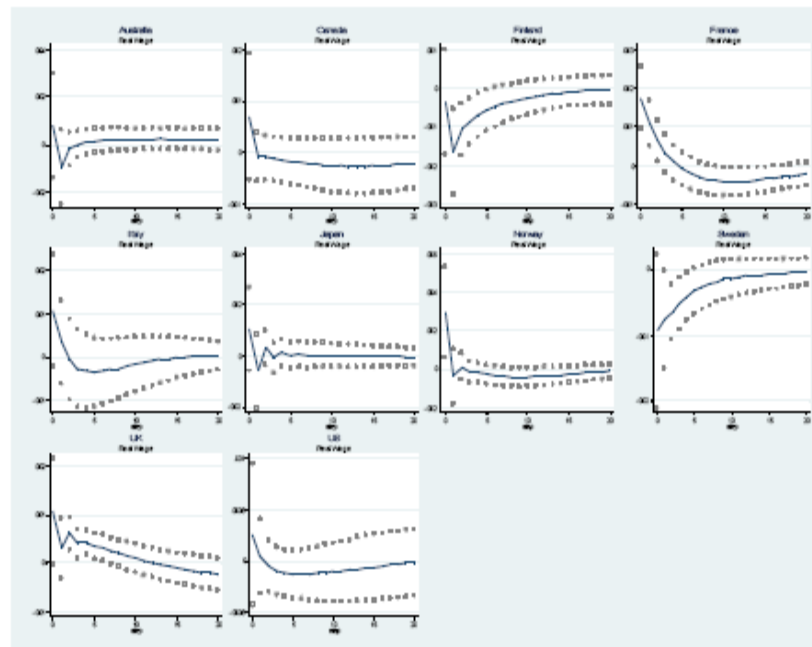


Figure 8. The Effect of Government Expenditure Shocks on the Labor Force Participation Rate

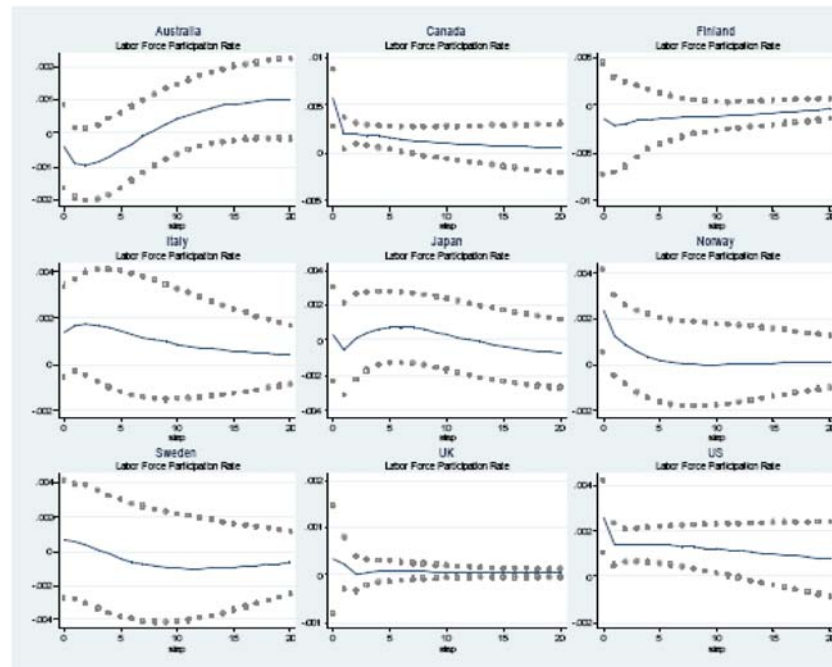
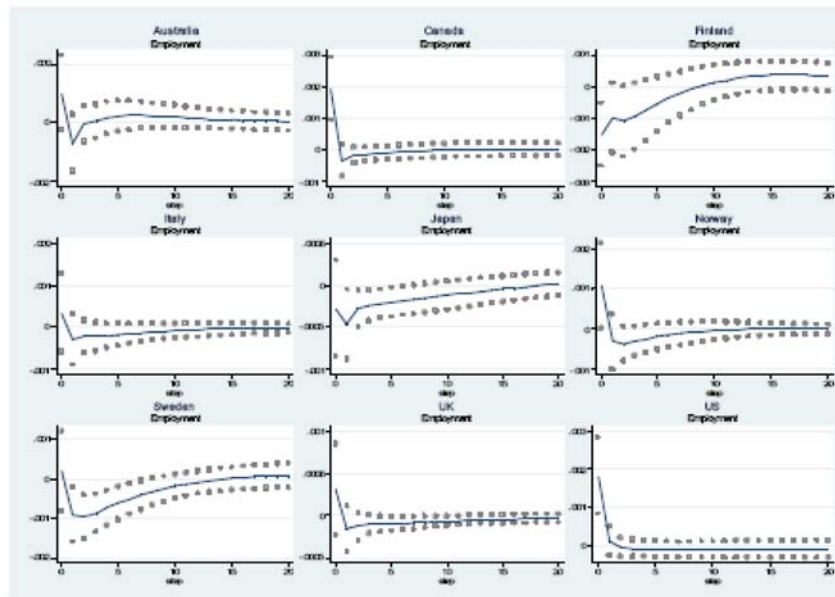


Figure 9. The Effect of Government Expenditure Shocks on Employment



# Robustness:

Figure 3. The Effect of Government Expenditure Shocks on the Unemployment Rate (Ramey-Shapiro War Dummy Approach)

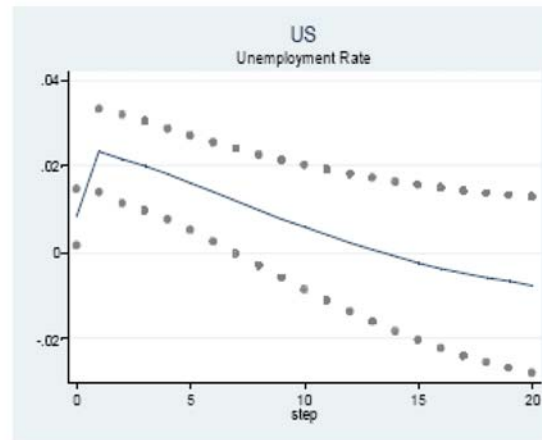
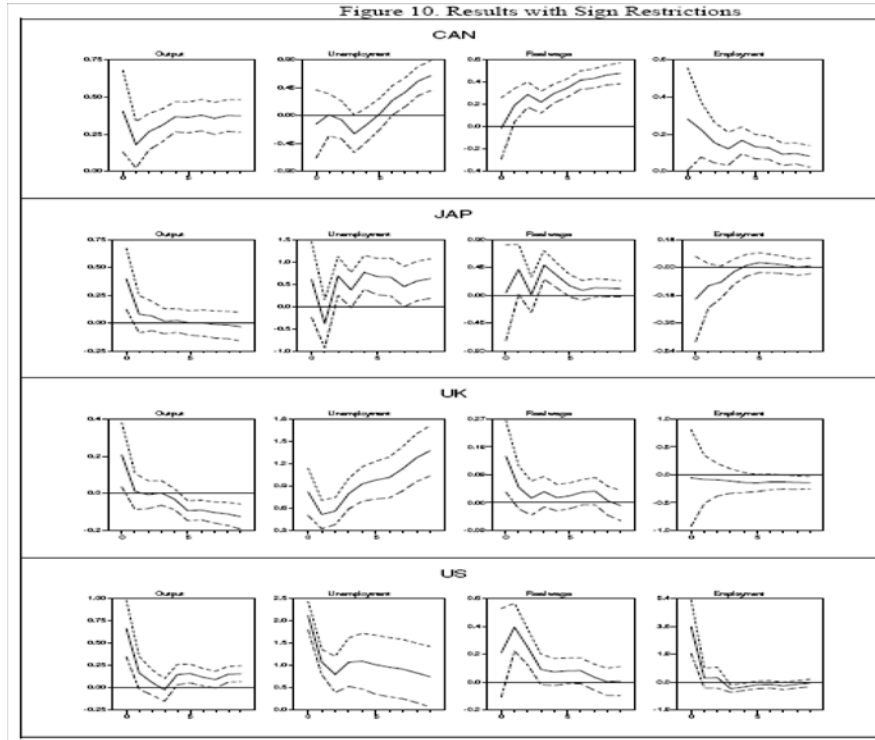


Figure 10. Results with Sign Restrictions



## Summary:

- Fiscal expansions tend to increase employment, the real wage, output, the labor force participation and the unemployment rate for the majority of OECD countries
- Significant increase also in vacancy postings and insignificant increase in labor market tightness.
- Results are robust to alternative ways of identifying fiscal shocks.

## New features of the model

- Labor market participation decisions: trade-off between leisure and the returns from a match.
- Payoff from engaging in search activities is smaller for labor market non-participants (outsiders) than for search active agents (insiders)- they face an inferior matching technology.
- The rest is similar to the previous model

## Preferences

At the beginning of each period a fraction  $n_{t-1}$  of the household's members is employed, a fraction  $u_t$  is unemployed and a fraction  $l_t$  does not participate in the labor market.

$$1 = n_{t-1} + u_t + l_t \quad (25)$$

The preferences of the representative household are:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \phi \frac{l_t^{1-\zeta}}{1-\zeta} \quad (26)$$

where  $c_t = c_{e,t}n_t + c_{u,t}u_t + c_{l,t}l_t$  is consumption,  $1/\eta$  the intertemporal elasticity of substitution,  $\phi > 0$  is a preference parameter and  $\zeta$  is the inverse of the elasticity of labor supply.

- Households pool income of the members and share consumption.

## Matching

- Every period a constant fraction  $\rho$  of the current worker-job matches is destroyed and a measure of  $m$  new matches is formed.
- Terminated workers enter into unemployment. An insider may either remain unemployed, find a new job, or become an outsider. Insiders move to long term unemployment and become outsiders with probability  $\mu \in [0, 1]$ .
- Aggregate matches are:

$$\begin{aligned} m(v_t, u_t^O, u_t^I) &= m_I(v_t, u_t^I) + m_O(v_t, u_t^O), & \text{with} & & (27) \\ m_I(v, u) &\geq m_O(v, u) & \text{for } \forall v, u > 0 & & \end{aligned}$$

where  $v$  are vacancies,  $u^I$  is the measure of insiders in unemployment,  $u^O$  is the measure of outsiders looking for a job.

- Assume

$$m_j(v, u^j) = \gamma_m^j v^{1-\gamma} (u^j)^\gamma \quad j = I, O, \quad \gamma_m^I > \gamma_m^O > 0 \quad (28)$$

- The labor market tightness is  $\theta_t = \frac{v_t}{u_t}$  and it is a function of the relative size of insiders and outsiders.

- The probability that a vacant job is filled is:

$$q_t = \frac{m_t}{v_t} = \theta_t^{-\gamma} \left[ \gamma_m^I \left( \frac{u_t^I}{u_t} \right)^\gamma + \gamma_m^O \left( \frac{u_t^O}{u_t} \right)^\gamma \right] \quad (29)$$

where  $u_t = u_{It} + u_{Ot}$ , and  $\frac{u_t^j}{u_t}$ ,  $j = I, O$ , defines the share of unemployment for the each type of agent.

- The higher the unemployment rate for both type of agents, the lower the probability that a vacancy will be filled. An increase in the unemployment rate for insiders has a stronger impact on this probability than an increase in the unemployment rate of outsiders.
- The probability for an unemployed worker (insider or outsider) to find a job is:

$$p_t = \frac{m_t}{u_t} = \theta_t^{1-\gamma} \left[ \gamma_m^I \left( \frac{u_t^I}{u_t} \right)^{1-\gamma} + \gamma_m^O \left( \frac{u_t^O}{u_t} \right)^{1-\gamma} \right] \quad (30)$$

- The probabilities for an outsider (insider) to find a job are:

$$p_{ji} = \frac{m_{jt}}{u_t}, \quad j = O, I \quad (31)$$

- The employment transition equation is:

$$n_t = (1 - \rho)n_{t-1} + m_{It} + m_{Ot} \quad (32)$$

The transition equation for unemployment among insiders is given by:

$$u_{t+1}^I = (1 - \mu)u_t^I + \rho n_t - m_{It} \quad (33)$$

- The search trade-offs of insiders and outsiders differ.

## The problem of the household

The capital stock evolves over time according to:

$$k_{t+1} = (1 - \delta)k_t + i_t + \frac{\omega}{2} \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 k_{t+1} \quad (34)$$

where  $\delta$  is a depreciation rate,  $i_t$  gross investment and  $\omega$  a parameter.

The representative household maximizes:

$$\max E_t \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \quad (35)$$

choosing  $c_t$ , the number of unemployed insiders in the next period,  $u_t^I$ , and the number of unemployed outsiders,  $u_t^O$ , employment  $n_t$ ,  $B_{t+1}$ ,  $k_{t+1}$ ,

subject to (25), (32), (33), (34) and

$$c_t + i_t + \frac{B_{t+1}}{p_t R_t} \leq r_t k_t + w_t n_t + b u_t + \frac{B_t}{p_t} + \Pi_t - T_t \quad (36)$$

where  $p_t$  is the price level,  $w_t$  is the real wage,  $r_t$  is the real return to capital,  $b$  are unemployment benefits,  $R_t$  is the gross nominal interest rate,  $\Pi_t$  are the profits of the monopolistic competitive firms and  $T_t$  are government transfers.

## Intermediate good firms and job creation

The production function for intermediate goods is:

$$y_t = F(k_t, n_t) = k_t^\alpha n_t^{1-\alpha} \quad (37)$$

Problem: given  $n_{t-1}$  currently employed workers, choose capital and vacancies to maximize:

$$Q(k_t, n_{t-1}) = \max x_t F(k_t, n_t) - w_t n_t - r_t k_t - \kappa v_t + E_t \Lambda_{t+1} Q(k_{t+1}, n_t) \quad (38)$$

where  $x_t$  is the relative price of intermediate goods and  $\Lambda_{t+s}$  the discount factor.

- Available employment:

$$n_t = (1 - \rho_t) n_{t-1} + q_t v_t \quad (39)$$

## Bargaining

Workers and firms split surplus through Nash bargaining: for  $\vartheta \in (0, 1)$

$$\max_{w_t} (1 - \vartheta) \ln V_t^W + \vartheta \ln V_t^F \quad (40)$$

$V_t^W = w_t - b + (1 - \rho - (\psi_t^{Ih} + \psi_t^{Oh})) E_t \Lambda_{t+1} V_{t+1}^W$ , is the worker's surplus  
 and  $V_t^F = x_t(1 - \alpha) \frac{y_t}{n_t} - w_t + \beta E_t \Lambda_{t+1} V_{t+1}^F$ , is the firm's surplus,  $\psi_t^{Ih} = \frac{m_t^I}{u_t^I}$   
 and  $\psi_t^{Oh} = \frac{m_t^O}{u_t^O}$ .

Contractual wage:

$$w_t = (1 - \vartheta) \left[ (1 - \alpha) x_t \frac{y_t}{n_t} + \frac{\kappa(\psi_t^{Oh} + \psi_t^{Ih})}{q_t} \right] + \vartheta b \quad (41)$$

- The wage paid to matched unemployed insiders/ outsiders is the same.  
(Realistic? Problem?)

## Retailers and price setting

- A continuum of monopolistically competitive retailers indexed by  $i$ . Final goods is the composite of individual retail goods:

$$y_t = \left[ \int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (42)$$

$\varepsilon > 1$  is the constant elasticity of demand. Price of retail good is  $p_t = \left( \int_0^1 p_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ .

- Demand for good  $i$

$$y_{it} = \left( \frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t \quad (43)$$

- Calvo price stickiness: each retailer can reset its price with a fixed probability  $1 - \chi$ . The price index is :

$$p_t = \left[ (1 - \chi)p_t^*{}^{1-\varepsilon} + \chi p_{t-1}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \quad (44)$$

The optimal price solves:

$$p_{it}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{t=0}^{\infty} \chi^s \Lambda_{t+s} x_{t+s} y_{it+s}}{E_t \sum_{t=0}^{\infty} \chi^s \Lambda_{t+s} y_{it+s}} \quad (45)$$

## Fiscal and Monetary policies

$$bu_t + G_t = T_t \quad (46)$$

$$R_t = \bar{R} \exp(\zeta_\pi \pi_t) \quad (47)$$

where  $b$  is unemployment compensation and  $\pi_t$  is inflation in deviation from the steady state.

### Aggregate constraint

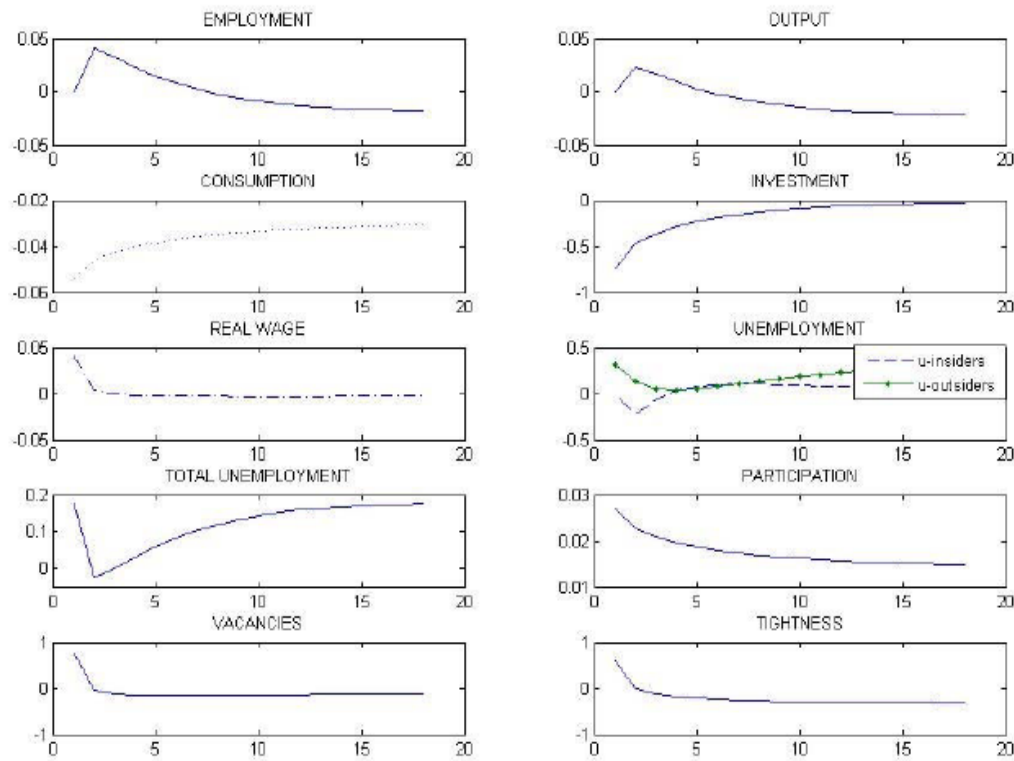
$$y_t = c_t + i_t + G_t + \kappa v_t \quad (48)$$

## 4.1 Dynamics

Parameter choice:

- $\beta = 0.99$ ,  $\sigma = 2$ ,  $\zeta = 4$ .
- $\gamma = 0.6$ ,  $\gamma = \vartheta$ .
- $\rho = 0.09$  following Davis, Haltiwanger and Schuh (1996) and Hall (1995).
- The probability of moving from short to long term unemployment  $\mu = 0.1$  (long term unemployment is 21% of total unemployment in the US).
- $\gamma_m^O$   $\gamma_m^I$  are set so that the total unemployment rate and the market tightness are equal 7% and 0.25, respectively.

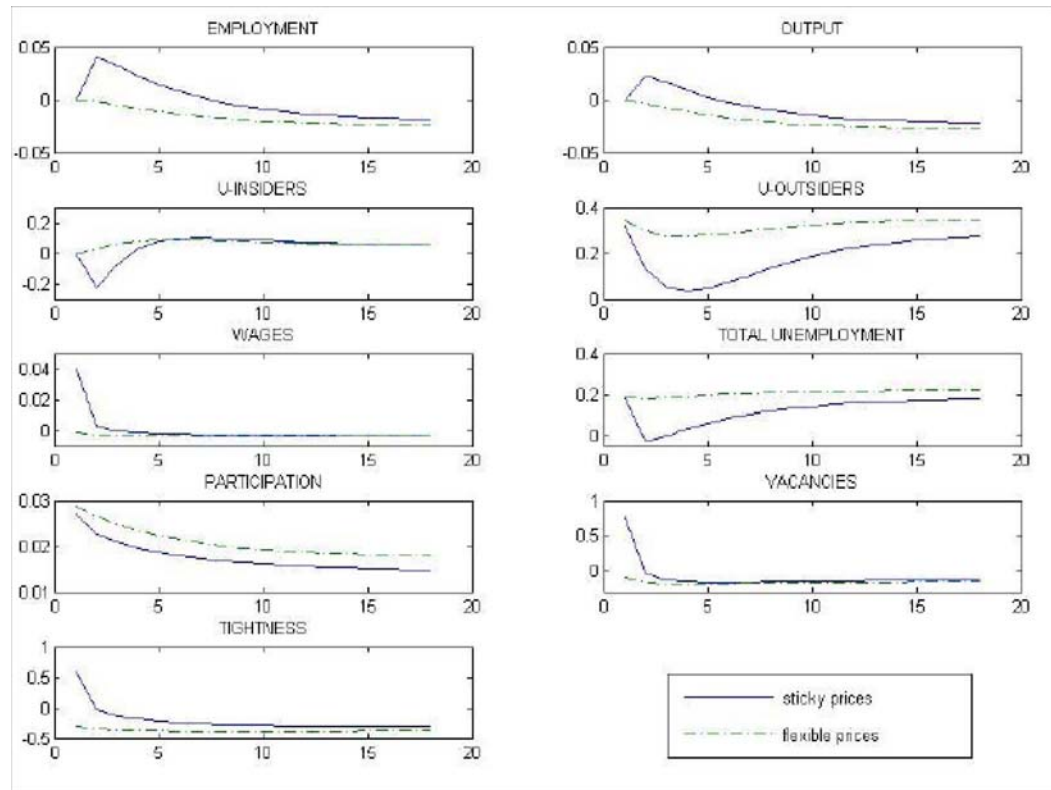
- The level of benefits  $b$  is set so that labor force participation in the steady state is 70% . The vacancy to output ratio is set equal to 0.01.
- $\delta = 0.01$  and  $\alpha = 0.36$ .  $\omega$  is set to match the ratio of the investment to output variance for the US when we include TFP and monetary shocks in the model.  $\chi$  is set to 0.75,
- Government spending process taken from the data.



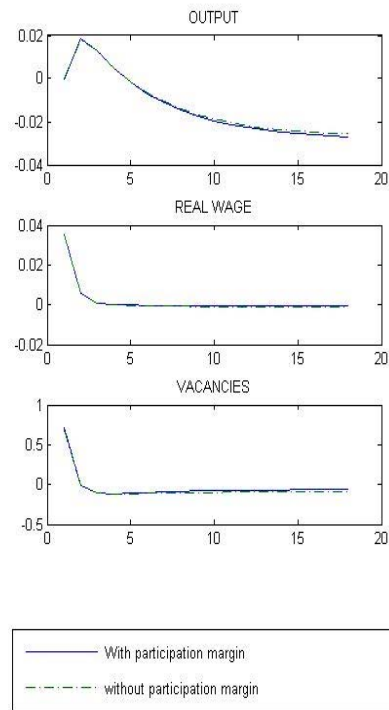
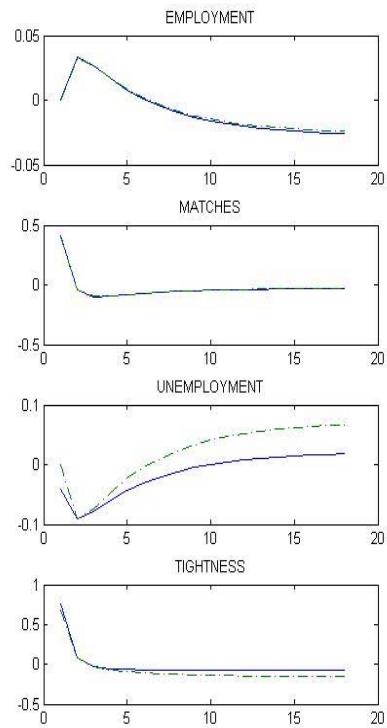
- Higher government spending makes labor supply and the participation

increase.

- Crowds out private consumption and investment.
- For output to increase, need to increase labor demand, wages and employment.
- Insiders get the extra jobs, outsiders' unemployment rate increases.
- Total unemployment up on impact because of the increase in participation and the increase in the unemployment rate of outsiders. As insiders are hired by the firms, unemployment decreases, but when the demand effect fades away total unemployment starts rising again.
- Unemployment effects of the shocks are very persistent. Hysteresis?



- With flexible prices, the increase in government absorption crowds out vacancy posting.
- Although the wealth effect would increase participation and the labor supply in equilibrium, the decrease in vacancy posting would decrease demand for labor. Thus output falls and the unemployment of both types of agents increases.



## Participation vs non-participation margin, no heterogeneity

- If only labor force participation is added: unemployment falls.
- Workers heterogeneity is crucial for generating the increase in total unemployment after the spending shock for low values of the labor supply elasticity.
- If agents were homogeneous, an increase in government spending under sticky prices would increase labor demand and unemployment would be reduced.
- It is the fact that outsiders have a hard time to find a job that makes total unemployment increase in equilibrium when the labor supply elasticity is low.
- When labor supply elasticity is high, unemployment always decreases.

## Interesting questions

- What happens if the efficiency of the matching technology for outsiders is a function of the duration of non-participation (for example, exponentially declining)?
- What if the utility function differentiates between being a participant or a non-participant (so that  $U(l_t) \neq -U(n_t + u_t)$ )?
- What if benefits  $b$  are functions of the duration of unemployment (for example, exponentially declining)?
- What if there are only distorting taxes?