Conflict Networks*

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Abstract

We analyze a model of multiple bilateral conflicts embedded in a network structure where opponents invest in specific conflict technology to attack their respective rivals and defend their own resources. For a general specification of this conflict game there exists a unique equilibrium where each opponent invests positive amounts into conflict technology directed against all its respective rivals. Total conflict intensity, measured as aggregated equilibrium investment into all bilateral conflicts, depends on the underlying network characteristics. We analyze three specific but general classes of conflict structures (regular, star-shaped, and bipartite conflict networks) and show that peaceful resolution of bilateral conflicts, interpreted as dissolving conflictive links, induces a decrease in conflict intensity for the conflict classes considered. Additionally, a negative relation between individual conflict investment and network centrality can be established for those classes. Extending the analysis to general irregular networks illustrates the limits of this approach because peaceful bilateral conflict resolution might in some cases induce an increase in conflict intensity. The derived results have implications for peaceful resolutions of conflicts because neglecting the fact that opponents are embedded into a interrelated conflict structure might have adverse consequences for conflict intensity.

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1 Introduction

Violent conflicts and wars are frequently observed phenomena in human history and have always been in the research focus of social scientists. Recently, the economists profession became increasingly interested in establishing general, and therefore abstract models of conflicts based on a game-theoretical setup with rational and strategic players. Most of this growing literature is concentrated on the analysis of stylized models of singular or isolated conflicts based on the specific idiosyncratic characteristics of the involved conflict parties, e.g., Esteban and Ray (2008), Jackson and Morelli (2008), Caselli and Coleman (2006), Basu (2005), and Beviá and Corchón (2008). Our approach should be considered supplementary to this literature in the sense that we add a structural element to the analysis: In our model each conflicting party faces more than a singular and isolated conflict, i.e., the opponents are embedded into a hostile but structured environment of conflictive relations. This structure of bilateral conflicts is interpreted as a fixed and given network where the conflict parties (which can be individuals, social entities, countries, etc.) are represented by the nodes of a network while the bilateral conflicts are interpreted as the links between the respective nodes. Hence, we interpret the structure of interrelated conflicts as a simultaneously played conflict game, consisting of several distinctive conflicts, played on a fixed and given network.

In our approach, we assume that the outcome of each singular bilateral conflict depends on the investment into conflict specific technology by the conflicting parties, for instance, military equipment, mercenaries, etc. The investment decision of each party (that will act strategically) depends not only on the investments of the respective direct rivals but also (indirectly) on the investment decision of the rivals of their rivals which induces a feedback effect that is common in network models of social interaction.[1] We model each bilateral conflict as a transfer contest, where contested resources are transfered from the winner to the looser, see Appelbaum and Katz (1986), Hillman and Riley (1989), and Leininger (2003). This implies that conflict investment is socially inefficient with respect to aggregated expected equilibrium effort. However, even if not investing in a specific bilateral conflict is the preferred outcome by both opponents, they cannot commit to this strategy if there exists a conflictive relation between them.

In this study we are especially interested in the relation between conflict intensity (measured as the total equilibrium investment into conflict technology by all opponents) and the underlying network structure. This analysis is carried out by considering the following three classes of conflict structures, that are related to historical examples including, for instance, the Western Roman

[1] The model is formulated in general terms such that different interpretations for the underlying structured environment are possible: for instance, lobbying of several firms for several distinctive issues at different authorities could also be interpreted as a network of bilateral conflicts where two firms are connected if they lobby for the same issue. Analyzing the dependence of overall lobbying activity on the underlying relations of lobby issues and firms is an important issue due to the social waste that is generated by these activities.

Empire and the Greek city states of the Mycenaean civilization:

- Regular conflict networks that are characterized by a large degree of symmetry among the opponents. We provide an historical example of conflictive peer polity interaction that can be represented as a regular conflict network.
- Star-shaped conflict networks that are characterized by a large degree of asymmetry between a center and its periphery. Historically, the multitude of conflictive relations among an empire and its surrounding neighbors has this kind of core-periphery structure.
- Complete bipartite conflict networks where two coalitions are in conflict against each other. Ideological conflicts can be interpreted as bipartite conflict networks because all members of one coalition share the same ideology and consider each member of the hostile coalition as a potential enemy.

For these classes of conflict networks there exists an intuitive relation between the underlying network characteristics and conflict intensity: In general, conflict intensity is increasing in the number of conflictive relations, depending on the relevant specifications of each class of considered conflict networks. These results are summarized by expressing the relation between conflict intensity and network characteristics in terms of a prominent centrality measure, i.e., eigenvector centrality, for the classes of networks mentioned above.

The relation between conflict intensity and network characteristics can also be stated out of the perspective of peaceful conflict resolution. Peaceful resolution of bilateral conflicts is here interpreted as resolving bilateral conflictive links within the conflict network. In a conflict network agents cannot commit to not invest into bilateral conflicts if they are in a conflictive relation among each other because there exists no commitment device. Our interpretation of peaceful conflict resolution in this setting is that of an exogenous commitment device that allows agents to commit to not engage in hostile activities, e.g., United Nations peacekeeping operations or observer missions. If theses operations are applied to a specific bilateral conflict then the opponents have a commitment device at their disposal and the bilateral conflict is peacefully resolved, i.e., the link ceases to exist. The natural question is then how conflict intensity is changed if bilateral conflicts are peacefully resolved, i.e., if conflictive links in the conflict network are cut. The mentioned result then implies that within each of the considered classes of conflict networks peaceful conflict resolution is beneficial because total conflict intensity is decreased if the number of links in the respective class is reduced.

However, extending the analysis to general irregular conflict structures shows the limit of this approach because peaceful resolution of bilateral conflicts might have adverse consequences if the conflict resolution process does not consider the underlying structure of related conflicts. We provide an example where conflict intensity is in fact increased as a consequence of conflicts resolution.

Additionally we characterize equilibrium behavior in general irregular networks indirectly (because no closed form solutions exist) that allows us to relate individual aggregate conflict investments with expected winning prospects of each agent.

Our approach is related to the recent network literature that considers games that are played on a fixed and given network structure, for instance, Bramoullé and Kranton (2007). An especially close relation exists to Ballester et al. (2006), where the consequences of the network structure on an aggregated measure of equilibrium actions are analyzed. There are, however, two important differences to our approach: firstly, we are interested in identifying crucial links within the network structure, i.e. bilateral conflict, instead of key players, and more importantly, we are not assuming that individual pay-off is governed by a linear quadratic utility function. Instead we apply a (in our framework more realistic) functional form that is well established in the literature on conflict analysis.[2]

The rest of the paper is structured as follows: In the next section we set up a general model of interrelated bilateral conflicts that can be represented as multiple contest games played on a fixed and given network. Heterogeneity among the opponents is induced endogenously by their (different) location in the network. We show that for this setup a unique equilibrium exists and define a notion of total conflict intensity as the aggregated investment into conflict specific technology. In section 3 we analyze the mentioned three classes of conflict structures and establish the positive relation between the number of bilateral conflicts and total conflict intensity, as well as the dependence between conflict intensity and eigenvector centrality of the classes considered. Moreover, an example outside the considered class of networks is provided where bilateral conflict resolution induces an increase in conflict intensity. In section 4 the analysis is extended to general irregular networks starting with a characterization of equilibrium behavior. Although there do not exist closed form solutions for this general case, partial results are provided that allow the ordering of bilateral conflicts according to the aggregated conflict spending that is invested into each of them. In section 5 some possible generalizations of the model are discussed and section 6 concludes.

2 The Model

There is a set $N = \{1, \ldots, n\}$ of conflicting parties, individuals, or any type of social entities (from now on called agents) that are embedded in a fixed structure of bilateral conflicts, i.e., each agent i is engaged into bilateral conflicts with some agents j where j is called a rival of agent i. The set of rivals of agent i is denoted by $N_i \subseteq N \setminus i$ and each member of N_i can initiate a bilateral conflict

[2]However, this makes our analysis more complex because the pay-off function is non-monotonic which implies that closed form solutions for general irregular networks do not exist. Additionally, the non-monotonicity of our model implies that it cannot be subsumed into the rather general class of network games with strategic substitutes or complements, as defined, for instance, in Galleotti et al. (2007).

against i such that agent i faces $n_i = |N_i|$ potential bilateral conflicts. This underlying structure of bilateral conflicts can be interpreted as a fixed and given network which is represented by a graph $\mathbf{g}$ consisting of nodes and links between them. The nodes of graph $\mathbf{g}$ correspond to the agents embedded into a network of conflictive relations that is formed by the links between nodes that represent potential bilateral conflicts. Hence, if agent i is in conflict with opponent j then $g_{ij} = 1$, while if there is no conflictive relation between them then $g_{ij} = 0$. It is assumed that both opponents in a bilateral conflict are affected in the same way, i.e., the network is undirected and symmetric: $g_{ij} = g_{ji}$. The set N_i of rivals of agent i can then be defined as $N_i = \{j \in N \setminus i : g_{ij} = 1\}$.

The outcome of each bilateral conflict is probabilistic and depends on the investment into conflict specific technology by the respective rivals. The investment of agent i into the conflict against rival $j \in N_i$ is denoted by $e_{ij} \in \Re_+$ and the n_i-dimensional vector of conflict spendings of agent i against all her rivals (the strategy of agent i) by $\mathbf{e_i} = \{e_{ij}\}_{j \in N_i}$. The vector of conflict spending that is directed against agent i by all of her respective rivals is denoted by $\mathbf{e_{-i}} = \{e_{ji}\}_{j \in N_i}$.

It is assumed that each bilateral conflict is zero-sum with respect to the contested resources, i.e., the loser has to compensate the winner in the sense that an amount V of resources of the loser are transfered to the winner.[3] This assumption reflects the frequently observed fact that underlying motivations for conflict are contested natural resources, or territory, and that looting is and was a frequently observed behavior of the winning conflict party.[4]

In addition, it is assumed that rivals have identical perceptions with respect to potential gains and losses in each bilateral conflict in which they are engaged, e.g., if agent i wins the conflict against any of her rivals $j \in N_i$ she obtains an amount V of resources of rival j, if agent i looses against j an amount V of her own resources are transferred to the winning agent j, and vice versa for all agents j.[5] This implies that all bilateral conflicts are symmetric for the respective rivals. We make this symmetry assumption because our main interest is the effect of the network structure that by itself already induces some heterogeneity on the agents.

The outcome of each bilateral conflict is governed according to a probability function that maps the conflict specific investments of the respective two opposing rivals into a probability to win the respective conflict, i.e., agent i wins the bilateral conflict against rival j with probability $p_{ij} = p(e_{ij}, e_{ji}) \in [0, 1]$, which is differentiable, increasing and concave in own spendings e_{ij}. As $p_{ij} = 1 - p_{ji}$, this probability function is also decreasing and convex in the spending e_{ji} of the respective rival. It is assumed that this probability is symmetric in the sense

[3]This kind of 'transfer contest' is strategically equivalent to a specification where the loser of the conflict does not have to pay anything and the winner gets a prize of $2V$, see Eq. (1).

[4]In Collier and Hoeffler (2004), for instance, it is shown that economic factors ('greed'), like primary commodities and opportunity costs, have more predictive power for the outbreak of civil war than political factors ('grievance'), e.g. inequality or ethnic polarization.

[5]This assumption implies that agents that have more rivals might potentially gain more but also loose more resources than agents with a lower number of hostile neighbors.

that if two rivals i and j spend the same amount $e = e_{ij} = e_{ji}$ they will win the conflict with the same probability: $p(e, e) = \frac{1}{2}$. Together with the symmetry assumption this also implies (in the case $e = e_{ij} = e_{ji}$) that the expected value of obtained (or lost) resources is zero for both agents.

Spending in conflict against rivals is related with a cost $c(\mathbf{e_i})$ that is an increasing and strictly convex function with $c(0, \ldots, 0) = 0$.

The expected payoff function of agent i is additively separable in costs and expected wins and losses of all bilateral conflicts in which she participates, and can be stated in the following way:

$$\pi_i(\mathbf{e_i}, \mathbf{e_{-i}}; \mathbf{g}) = \sum_{j \in N_i} p_{ij} V + \sum_{j \in N_i} (1 - p_{ij})(-V) - c(\mathbf{e_i}).$$

For notational simplicity we reformulate this expression as follows:

$$\pi_i(\mathbf{e_i}, \mathbf{e_{-i}}; \mathbf{g}) = W(\mathbf{e_i}, \mathbf{e_{-i}}) - c(\mathbf{e_i}), \tag{1}$$

where $W(\mathbf{e_i}, \mathbf{e_{-i}}; \mathbf{g}) = 2V \sum_{j \in N_i} p_{ij} - n_i V$ denotes the expected 'revenue' of conflict for agent i, i.e., the aggregated expected amount of transfered resources that agent i wins or looses in all bilateral conflicts in which she is involved. Note that, due to the fact that each conflict is modeled as a transfer contest where losers have to compensate the winner, the total expected revenue of the overall conflict game (or, in other words, the aggregated value of contested and transfered resources) is zero independently of the network structure:

$$\sum_{i \in N} W(\mathbf{e_i}, \mathbf{e_{-i}}; \mathbf{g}) = 0. \tag{2}$$

This implies that equilibrium behavior does purely depend on the strategic response to the network structure and is not confounded by the fact that different network structures induce different values of aggregated resources.

The setup as stated above has the structure of a concave n-person game in the sense of Rosen (1965). In this paper a proof for existence and uniqueness of equilibrium is provided that is based on an additional condition with respect to the concavity of the payoff functions, which is called diagonal strict concavity. We use this result to show that a unique equilibrium exists in our conflict game[6] because our setup satisfies this additional condition. The additional restriction is formulated with respect to the following function, a weighted sum of the payoff functions of all agents:[7]

$$\sigma(\mathbf{e}, \mathbf{r}) = \sum_{i \in N} r_i \, \pi_i(\mathbf{e_i}, \mathbf{e_{-i}}), \tag{3}$$

[6]The conflict game is neither an aggregative game (the reaction functions cannot be expressed in terms of the aggregate of the strategies of all the other players), nor a supermodular game (because the reaction functions are non-monotonic) which implies that common existence proofs that are based on those characteristics are not applicable in this setup.

[7]For reasons of notational simplicity the dependence on graph $\mathbf{g}$ is suppressed in the following paragraphs.

where $\mathbf{e} = (\mathbf{e_1}, \ldots, \mathbf{e_n})$, $\mathbf{r} = (r_1, \ldots, r_n)$, and $r_i \geq 0$. Theorem 1 and 2 of Rosen (1965) provide the following sufficient condition for the existence of a unique equilibrium for the case of an orthogonal constraint set[8]: the function $\sigma(\mathbf{e}, \mathbf{r})$ must be diagonally strictly concave. Using a characterization of Goodman (1980), the following conditions on the payoff functions are equivalent to a diagonally strictly concave function $\sigma(\mathbf{e}, \mathbf{r})$:

(i) $\pi_i(\mathbf{e_i}, \mathbf{e_{-i}})$ is strictly concave in $\mathbf{e_i}$,

(ii) $\pi_i(\mathbf{e_i}, \mathbf{e_{-i}})$ is convex in $\mathbf{e_{-i}}$, and

(iii) $\sigma(\mathbf{e}, \mathbf{r})$ is concave in $\mathbf{e}$ for some $\mathbf{r}$ with $r_i > 0$ for all $i \in N$.

We will follow this approach and show that our conflict game satisfies all three conditions which implies that $\sigma(\mathbf{e}, \mathbf{r})$ is diagonally strictly concave which is sufficient for existence and uniqueness of equilibrium.

Proposition 1 *There exists a unique equilibrium in the conflict game.*

Proof. The conditions for a diagonally strictly concave function $\sigma(\mathbf{e}, \mathbf{r})$ are:

(i) $\pi_i(\mathbf{e_i}, \mathbf{e_{-i}})$ is strictly concave in $\mathbf{e_i}$.

$W(\mathbf{e_i}, \mathbf{e_{-i}})$ is concave in $\mathbf{e_i}$ for all $i \in N$ because its Hessian is negative semi-definite, i.e., it is a (diagonal) matrix with entries $\frac{\partial^2 W(\mathbf{e_i}, \mathbf{e_{-i}})}{\partial e_{ij}^2} = \frac{\partial^2 p_{ij}}{\partial e_{ij}^2} \leq 0$ on the diagonal (by the assumption on concavity of the probability function with respect to own conflict spending) and $\frac{\partial^2 W(\mathbf{e_i}, \mathbf{e_{-i}})}{\partial e_{ij} \partial e_{ik}} = \frac{\partial^2 p_{ij}}{\partial e_{ij} \partial e_{ik}} = 0$ elsewhere for all $j, k \in N_i$ with $j \neq k$. As $c(\mathbf{e_i})$ is assumed to be strictly convex for all $i \in N$, the payoff function $\pi_i(\mathbf{e_i}, \mathbf{e_{-i}})$ is the sum of a constant, a concave and a strictly concave function in $\mathbf{e_i}$. Hence, it is strictly concave in $\mathbf{e_i}$ for all $i \in N$.

(ii) $\pi_i(\mathbf{e_i}, \mathbf{e_{-i}})$ is convex in $\mathbf{e_{-i}}$.

$W(\mathbf{e_i}, \mathbf{e_{-i}})$ is convex in $\mathbf{e_{-i}}$ for all $i \in N$ because its Hessian is positive semi-definite, i.e., it is a (diagonal) matrix with entries $\frac{\partial^2 W(\mathbf{e_i}, \mathbf{e_{-i}})}{\partial e_{ji}^2} = \frac{\partial^2 p_{ij}}{\partial e_{ji}^2} \geq 0$ on the diagonal (by the assumption on convexity of the probability function with respect to conflict spending of the rival) and $\frac{\partial^2 W(\mathbf{e_i}, \mathbf{e_{-i}})}{\partial e_{ji} \partial e_{ki}} = 0$ elsewhere for all $j, k \in N_i$ with $j \neq k$. As the cost function does not depend on conflict spending of the rivals, the function $\pi_i(\mathbf{e_i}, \mathbf{e_{-i}})$ is convex in $\mathbf{e_{-i}}$.

(iii) $\sigma(\mathbf{e}, \mathbf{r})$ is concave in $\mathbf{e}$ for some $\mathbf{r}$.

Assume that $r_i = r > 0$ for all $i \in N$. Then Eq. (3) simplifies substantially due to the fact that the aggregated value of contested resources is zero, compare Eq. (2):

$$\sigma(\mathbf{e}, \mathbf{r}) = -r \sum_{i \in N} c(\mathbf{e_i})$$

[8] A constraint set is orthogonal if it is uncoupled. This is the case in the conflict game because the strategy space of each individual does not depend on the strategies of her rivals. Note also that in Rosen (1965) the strategy space for each $i \in N$ is convex and compact, which is, in principle, not the case in the conflict game defined above (here it is the non-negative orthant). However, we can construct a (sufficiently high) upper limit $\bar{e}$ such that all strategies $e_{ij} > \bar{e}$ are strictly dominated (for instance by choosing $e_{ij} = 0$) due to the fact that $W(\mathbf{e_i}, \mathbf{e_{-i}}) \in [-n_i V, n_i V]$ is bounded while $c(\mathbf{e_i})$ is unbounded. Hence, without loss of generality we can restrict attention to the strategy space $[0, \bar{e}]^{n_i}$ of non-dominated strategies for each individual $i \in N$ which is convex and compact.

By assumption, the cost function is strictly convex in own conflict spending. Hence, the function $\sigma(\mathbf{e}, \mathbf{r})$ is a sum of strictly concave functions which is also strictly concave.

As the three conditions are satisfied, function $\sigma(\mathbf{e}, \mathbf{r})$ is diagonally strictly concave which is a sufficient condition for the existence of a unique equilibrium of the conflict game. ■

Our main objective is the analysis of the overall conflict intensity, denoted by $E^*(\mathbf{g})$, and formally defined as the aggregated level of conflict spendings in equilibrium by all agents in all bilateral conflicts:

$$E^*(\mathbf{g}) = \sum_{i \in N} E_i^*(\mathbf{g}) = \sum_{i \in N} \sum_{j \in N_i} e_{ij}^*(\mathbf{g}),$$

where $E_i = \sum_{j \in N_i} e_{ij}$ is the aggregated conflict spending of agent i against all her rivals $j \in N_i$. The variation of the conflict intensity for different networks can then be determined by analyzing how $E^*(\mathbf{g})$ depends on the variables that characterize the network structure.

The following functional form for the pay-off function is specified that satisfies the assumptions made before and allows us to derive closed form equilibrium expressions for some classes of conflict structures.[9] For the rest of the paper we adopt the following quadratic specification of the cost function:

$$c(\mathbf{e_i}) = c(E_i) = (E_i)^2 = (\sum_{j \in N_i} e_{ij})^2. \tag{4}$$

The specific functional form of the probability function will be in the style of Tullock (1980) which is frequently applied in models of conflict and contests:[10]

$$p_{ij} = \begin{cases} \frac{e_{ij}}{e_{ij}+e_{ji}} & \text{if } e_{ij} + e_{ji} > 0, \\ 1/2 & \text{if } e_{ij} + e_{ji} = 0. \end{cases} \tag{5}$$

Note that this functional form does not fit exactly the setup as introduced before because of its discontinuity at point $(0, 0)$, i.e., in the case that two rivals do not spend anything in the respective bilateral conflict. However, this will never occur in equilibrium as it is shown in the following proposition:

Proposition 2 *For the conflict game specified by Eq. (1), (4) and (5) the unique equilibrium is interior.*

Proof. Assume that for all bilateral conflicts at least one rival exerts positive conflict spending. By Proposition 1 there exists a unique equilibrium. Relaxing now this assumption, i.e., allowing also strategies at the lower boundary of the strategy space, the following two claims have to be verified to sustain the derived

[9] In section 5 the relevance of the specific functional form for the derived results is discussed.

[10] Recent surveys that review the literature that applies this functional form are Corchon (2007) and Konrad (2007) for models of contests, as well as Garfinkel and Skaperdas (2006) for conflict models.

equilibrium: first, it cannot be an equilibrium to choose a strategy at the lower boundary of the strategy space, and second, strategies at the lower boundary are dominated by the interior equilibrium strategies.

Assume by contradiction that there exists an equilibrium $(\mathbf{e}_\mathbf{i}^*, \mathbf{e}_{-\mathbf{i}}^*)$, where (at least) one agent i decides not to invest in a bilateral conflict: $e_{ij}^* = 0$ for some $j \in N_i$. Consider now the bilateral conflict between agent i and j. It must be the case that either $e_{ji}^* > 0$, or that $e_{ji}^* = 0$. However, $e_{ji}^* > 0$ cannot be part of an equilibrium strategy because any $e'_{ji} \in (0, e_{ji}^*)$ is a profitable deviation for agent j: As agent j wins the conflict against i for sure by spending a small positive amount in this conflict, we have $\pi_j((e_{j1}^*, \ldots, e'_{ji}, \ldots, e_{jn_j}^*), \mathbf{e}_{-\mathbf{j}}^*; \mathbf{g}) > \pi_j(\mathbf{e}_\mathbf{j}^*, \mathbf{e}_{-\mathbf{j}}^*; \mathbf{g})$. Therefore, $(e_{ij}^* = 0, e_{ji}^* > 0)$ cannot be part of an equilibrium strategy. Also the second possibility $(e_{ij}^* = 0, e_{ji}^* = 0)$ cannot be part of an equilibrium strategy because $e'_{ji} = \epsilon$ for ϵ sufficiently small is a profitable deviation as $\pi_j((e_{j1}^*, \ldots, e'_{ji}, \ldots, e_{jn_j}^*), \mathbf{e}_{-\mathbf{j}}^*; \mathbf{g}) > \pi_j(\mathbf{e}_\mathbf{j}^*, \mathbf{e}_{-\mathbf{j}}^*; \mathbf{g})$ because $p(e'_{ji}, e_{ij}^*) = 1 > p(e_{ji}^*, e_{ij}^*) = 1/2$ due to the discontinuity at $(0,0)$ while $lim_{\epsilon \to 0} c(e_{j1}^*, \ldots, e'_{ji}, \ldots, e_{jn_j}^*) = c(\mathbf{e}_\mathbf{j}^*)$ in the limit because the cost function is continuous.

Hence, strategies at the lower boundary cannot be part of an equilibrium strategy, and strategies at the lower boundary are strictly dominated. Therefore all agents must spend positive amounts in the bilateral conflicts against all their respective rivals and the resulting equilibrium is interior. ∎

The fact that the equilibrium is interior also implies that the conflict game turns out to be a negative-sum game with respect to aggregated expected equilibrium payoff (by Eq. 2 the aggregated value of contested resources is zero) which captures the idea that conflicts are generally highly destructive and socially undesirable. In fact, the socially efficient outcome in this kind of conflict game (based on simultaneously played interrelated transfer contests) would be to not invest in conflict spending at all. However, as shown in the corollary, this outcome can never be the result of an equilibrium strategy as opponents cannot commit to refrain from investment in conflict spending if they are in a conflictive relation with each other.

The expected payoff function, based on Eq. (1), can then be expressed (where the discontinuity is suppressed in line with Proposition 2) in the following way:

$$\pi_i(\mathbf{e}_\mathbf{i}, \mathbf{e}_{-\mathbf{i}}; \mathbf{g}) = 2V \sum_{j \in N_i} \frac{e_{ij}}{e_{ij} + e_{ji}} - n_i V - (E_i)^2. \tag{6}$$

The related system of first order conditions for this specification of the payoff function has the following form:

$$\frac{e_{ki}^*(\mathbf{g})}{(e_{ik}^*(\mathbf{g}) + e_{ki}^*(\mathbf{g}))^2} V = E_i^*(\mathbf{g}), \qquad \text{for all } k \in N_i \text{ and all } i \in N. \tag{7}$$

This is a non-linear system with $\sum_{i \in N} n_i$ equations that does not allow a closed form solution for general conflict structures. Therefore we will concentrate our analysis on three classes of more specific conflict structures where closed form solutions of the above system can be derived. An analysis for general irregular networks follows in section 4.

3 Characteristic Classes of Conflict Networks

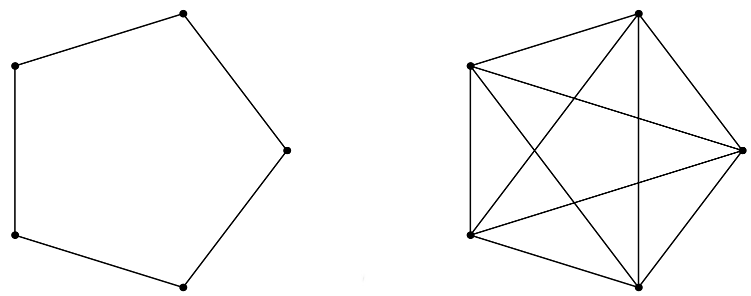
The following typology of conflict networks is based on their grade of symmetry. In our framework asymmetry is induced through the underlying network structure in the sense that agents with a high number of conflictive relations can potentially gain and also loose more resources than agents with less conflicts. Hence, we have as polar cases on one side highly symmetric conflict structures (regular conflict networks) where each agent has the same number of conflicts, while the other side consists of highly asymmetric conflict networks, e.g., conflicts among center and periphery. An intermediate class are complete bipartite conflict networks that are interpreted as ideological conflicts among two coalitions where members of one coalition are in conflict with each member of the opposed coalition. Although the conflict game is highly stylized we provide historical examples of conflicts that share the characteristics of the considered classes of conflict networks.

3.1 Regular Conflict Networks

Regular conflict networks are characterized by their high degree of symmetry among rivals. The symmetry property among opponents in a structured environment is also the crucial element in the concept of 'peer polities', a terminology used in historical anthropology. The concept of 'peer polity interaction' was introduced in Renfrew and Cherry (1986) to describe the historical fact that complex societies often developed through interaction of autonomous and homogenous social units that were not related to each other in forms of dominance and subordination. Peer polity interaction also included warfare and conflict. Historical examples that could be subsumed under this concept are, for instance, "the Mycenaean states, the later small city-states of the Aegean and the Cyclades, or the centers of the Maya Lowlands, that interact on an approximately equal level. [...] The evolution of such clusters of peer polities is conditioned not by some dominant neighbor, but more usually by their own mutual interaction, which may include both exchange and conflict", Tainter (1988), p. 201. Our focus is on hostile interaction among peer polities and we associate the symmetric nature of peer polity interaction with a regular conflict network.

Formally, a graph $\mathbf{g}$ is called *regular of degree* d if each agent $i \in N$ has the same number d of opponents: $n_i = d$ for all $i \in N$. A regular graph $\mathbf{g}$ is fully characterized by its degree d and the total number n of agents. The class of regular networks is denoted by R and incorporates cases such as the fully connected network, where $d = n - 1$, and a ring structure, where $d = 2$, compare figure 1.

Figure 1: Regular Conflict Structures: Ring (left) and Complete Network (right)



The following proposition describes the relation between those characteristics and conflict intensity for the class R of regular networks.

Proposition 3 *In regular conflict networks of class R conflict intensity is increasing in its degree d and in the total number n of agents in the network. Conflict intensity in a regular network R_1 is higher than in R_2 if and only if*

$$n_1\sqrt{d_1} > n_2\sqrt{d_2}.$$

Individual conflict investment and expected payoff in equilibrium is decreasing in d and does not depend on n. Moreover, expected equilibrium payoff is negative for all agents.

Proof. By Proposition 1 and Corollary 2 there exists a unique interior equilibrium of the conflict game. The following (symmetric) conflict investment $e^* \equiv e^*_{ij}$ for all $i \neq j$ solves the system of first order conditions; hence it must be the unique equilibrium:

$$e^*(R) = \frac{1}{2}\sqrt{\frac{V}{d}}, \qquad \text{for all } i \in N.$$

Total conflict intensity is defined as aggregated equilibrium spending:

$$E^*(R) = \sum_{i\in N}\sum_{j\in N_i} e^*(R) = n\,d\,e^*(R) = \frac{n}{2}\sqrt{d\,V}.$$

The last expression is increasing in the total number of agents and also in its degree d. Simplifying the inequality $E^*(R_1) > E^*(R_2)$ yields the displayed condition.

As the equilibrium is symmetric, the probability to win (or loose) each bilateral conflict is identical for all agents, i.e., $p^*_{ij} = \frac{1}{2}$ for all $i \neq j$. Using the obtained results to derive expected equilibrium payoff results in:

$$\pi(\mathbf{e}^*_\mathbf{i}, \mathbf{e}^*_{-\mathbf{i}}; R) = -d\frac{V}{4}, \qquad \text{for all } i \in N,$$

which is negative. It is also evident that $e^*(R)$ and $\pi(\mathbf{e_i^*}, \mathbf{e_{-i}^*}; R)$ are decreasing in d and independent of n which establishes the statements of the proposition. ■

In equilibrium all agents choose the same level of conflict investment which implies that they win each bilateral conflict with the same probability. As each bilateral conflict is a transfer contest, the sum of expected transfered resources must be equal to zero for each agent and in equilibrium the payoff is negative because agents also face the cost of conflict spending. This situation is socially (and also pareto) inefficient because universal peace would result in a payoff of zero. The fact that such a conflict structure induce socially inefficient results is also acknowledged in the historical analysis of the above mentioned examples: "successful competition by any Mycenaean polity would yield little real return. The result was probably constant investment in defense, military administration, and petty warfare, with any single polity rarely experiencing a significant return on that investment" (ibid., p. 204).

From the perspective of peaceful conflict resolution the question is how bilateral conflict resolution, i.e., dissolving links between agents, affects overall conflict intensity. Proposition 3 then implies that for the class of regular networks conflict intensity is reduced if the number of bilateral conflicts is decreased in the sense that either the degree of the network is decreased, or that agents are isolated such that they will not be part of the conflict network anymore.[11]

3.2 Star-Shaped Conflict Networks

After considering highly symmetry regular conflict networks we focus our attention now on asymmetric conflict structures that are star-shaped, i.e., where one agent is in conflict with all other potential rivals while none of the rivals is in conflict with each other, see the left part of figure 2. This class of conflicts has a center-periphery structure which is characteristic for historical empires that were frequently in permanent conflict with minor rivals at its periphery, for instance, the Western Roman Empire at the point of its largest expansion.

A star-shaped conflict network consists of an agent c in the center of the network who is connected with all other agents of the network such that $g_{ci} = 1$ for all $i \neq c$ and $n_c = n - 1$. All agents of set N_c at the periphery are only in conflict with the center but not with each other, $g_{ij} = 0$ for all $i, j \neq c$ and thus $n_i = 1$ for all $i \in N_c$. This implies that there are in total $n - 1$ bilateral conflicts in the star network. Hence, the class of star networks, from now on denoted by S, is completely characterized by $n_c = n - 1$, the number of agents in the periphery.

The payoff of the center agent c can be written as

$$\pi_c(\mathbf{e_c}, \mathbf{e_i}; S) = \sum_{i \in N_c} \frac{e_{ci}}{e_{ci} + e_{ic}} 2V - (E_c)^2 - (n-1)V, \tag{8}$$

[11] Note that the resulting conflict network must be a regular as well to be able to apply the result of the proposition. For a discussion of this issue compare section 3.

and the corresponding payoff by an agent p in the periphery is

$$\pi_p(e_{pc}, e_{cp}; S) = \frac{e_{pc}}{e_{cp} + e_{pc}} 2V - (e_{pc})^2 - V. \tag{9}$$

The following proposition summarizes the equilibrium relations in this class of conflict networks.

Proposition 4 *Conflict intensity in star-shaped conflict networks of class S is increasing in the number n_c of agents in the periphery.*

For the center agent individual (aggregated) conflict investment is decreasing (increasing) in n_c, while equilibrium probability and expected payoff is decreasing in n_c. For the periphery agent the same relation holds with respect to individual conflict investment, while the relation is inversed for equilibrium probability and payoff.

Proof. By Proposition 2 the equilibrium is interior and unique. Inspection of the first order conditions reveals that the center agent invests the same amount in each of her conflicts, i.e., $e_c^*(S) \equiv e_{ci}^*(S)$ for all $i \in N_c$. This also holds for each agent p in the periphery: $e_p^*(S) \equiv e_{jc}^*(S)$ for all $j \in N_c$. Calculating those expressions yields:

$$e_i^*(S) = p_i^*(S)\sqrt{V/\sqrt{n_c}} \text{ for } i \in \{c, p\}, \tag{10}$$

where $p_c^*(S) = \frac{1}{1+\sqrt{n_c}}$ and $p_p^*(S) = 1 - p_c^*(S)$ are the equilibrium probabilities for the center and the periphery agent to win a bilateral conflict. Note also, that $\frac{\partial p_c^*(S)}{\partial n_c} < 0$ and that $\frac{\partial p_p^*(S)}{\partial n_c} > 0$.

It can be shown that individual conflict investment as expressed in Eq. (10) is decreasing in n_c for the center agent c and the periphery agent p. Aggregated conflict investment of the center agent is $E_c^* = n_c\ e_c^* = \frac{n_c^{3/4}}{1+\sqrt{n_c}}\sqrt{V}$, which is increasing in n_c. Plugging the obtained expressions into the payoff functions in Eq. (8) and (9) yields the following expected equilibrium payoff:

$$\begin{aligned}
\pi_c(\mathbf{e_c^*}, \mathbf{e_p^*}; S) &= -\frac{n_c(\sqrt{n_c} + n_c - 1)}{(1+\sqrt{n_c})^2} V \\
\pi_p(e_p^*, e_c^*; S) &= \frac{n_c - 1 - \sqrt{n_c}}{(1+\sqrt{n_c})^2} V
\end{aligned}$$

Note that $\frac{\partial \pi_c(\mathbf{e_c}, \mathbf{e_p}; S)}{\partial n_c} < 0$ and that $\frac{\partial \pi_p(e_p, e_c; S)}{\partial n_c} > 0$. Hence, equilibrium payoff for the center agent is decreasing in n_c, while it is increasing for an agent at the periphery.

Finally, conflict intensity in a star conflict network can be expressed as:

$$E^*(S) = n_c[e_c^*(S) + e_i^*(S)] = n_c^{3/4}\sqrt{V}. \tag{11}$$

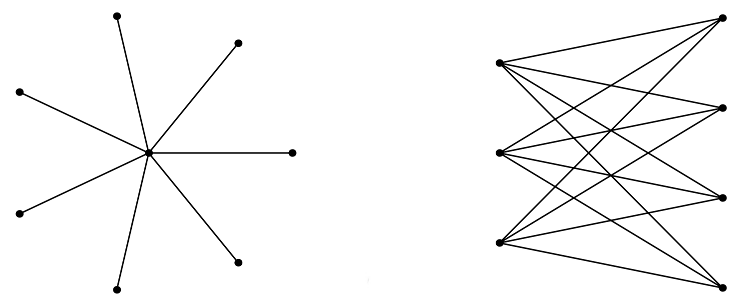
This expression is clearly increasing in n_c. ■

The results stated in the proposition imply that center agents loose if the number of bilateral conflicts with the periphery is increased. This is the case although it is beneficial for an agent to distribute the same amount of aggregated conflict investment on a greater number of bilateral conflicts because the probability function is concave in own conflict spending. However, the new opponents of the center agent also invest in the bilateral conflict which forces the center agent to increase total conflict investment as it is reflected by the fact the $E_c^*(S)$ is increasing in the number of agents in the periphery. However, this is not sufficient to induce equal or higher probability to win in each of her conflicts because $p_{ci}^*(S)$ is decreasing in n_c. In addition, for $n > 2$ we have that $p_{ci}^*(S) < \frac{1}{2}$. Hence, with more rivals the center will more frequently loose conflicts in expectation.

This has also consequences for expected equilibrium payoff $\pi_c^*(S)$ which is strictly decreasing in the number of rivals. Although this result should not be interpreted in the context of endogenous network formation, it provides an explanation for the historically observed tendency of expanding empires to collapse at some point in time because expansion requires more total investment for an increasing number of conflicts while at the same time this increase is not even sufficient to merely guarantee the status-quo. With respect to the mentioned historical example of the Western Roman Empire, "the economics of territorial expansion dictate, as a simple matter of mathematical probability, that an expanding power will ultimately encounter a frontier beyond which conquest and garrisoning are unprofitable. [...] The combined factors of increased costliness of conquest, and increased difficulty of administration with distance from the capital, effectively require that at some point a policy of expansion must end." (Tainter 1981, p. 148 f.)

The results derived above also imply that peaceful conflict resolution in star shaped conflict networks (resolving links between center and periphery) is beneficial because conflict intensity is decreased.

Figure 2: A Star-Shaped (left) and a Bipartite Conflict Network (right)



3.3 Complete Bipartite Conflict Networks

An intermediate case with respect to the symmetry of the underlying conflict structure are complete bipartite conflict networks where the members of two hostile coalitions are in conflict among each other. Hence, each agent is in conflictive relations with all the members of the hostile coalition, as represented in the right part of figure 2. This type of conflict structure resembles an ideological conflict because a member of one coalition perceives each member of the hostile coalition as an enemy and vice versa. Moreover, the common ideology among coalition members implies that there are no conflictive relations among agents of the same ideology. An historical example that fits to this description is the second world war where each country of the Axis Powers where (at least at some point in time) in conflict with nearly each member of the Allies.

A complete bipartite network, denoted by B, consists of two sets (coalitions) of agents, X and Y, that each have $|X| = x$ and $|Y| = y$ members. All members of set X are in conflict with each member of set Y and vice versa, such that $g_{ij} = 1$ for all $i \in X$ and all $j \in Y$. Agents of the same coalition share are not in conflict among each other: $g_{ij} = 0$ for all $i, j \in X$ or $i, j \in Y$.

The payoff function of an agent $i \in X$ can be stated as follows:

$$\pi_i(\mathbf{e_i}, \mathbf{e_{-i}}; B) = \sum_{j \in Y} \frac{e_{ij}}{e_{ij} + e_{ji}} 2V - (E_i)^2 - yV, \tag{12}$$

and vice versa for an agent that is member of coalition Y. Note also, that the star-shaped network is a special case of a bipartite network where one coalition only consists of one (center) agent. Therefore, the following proposition is a generalized version of proposition 4.

Proposition 5 *Conflict intensity in bipartite conflict networks of class B is increasing in the number x and y of each coalition. Conflict intensity in a complete bipartite network B_1 is higher than in B_2 if and only if:*

$$x_1\, y_1 > x_2\, y_2.$$

A large (relative) size of a coalition is beneficial for its members, i.e., each member of the more numerous coaltion invests less in in total conflict investment, wins each bilateral conflict with higher probability and has higher equilibrium payoff.

Proof. By Proposition 2 the equilibrium is interior and unique. Inspection of the first order conditions reveals that each member invests the same amount in each of her conflicts, e.g., for $i \in X$: $e^*_{ij}(B) = e^*_{ik}(B)$ for all $j, k \in Y$. This also holds across members of the same coalition, e.g., for $i, j \in X$: $e^*_{ik}(B) = e^*_{jk}(B)$ where $k \in Y$. Hence, there exist only two levels of individual conflict investment in a bipartite conflict network:

$$e^*_i(B) = p^*_i(B)\sqrt{V/\sqrt{x\, y}} \text{ for } i \in \{X, Y\}\ , \tag{13}$$

where $p_x^*(B) = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}$ for members of coalition X and $p_y^*(S) = 1 - p_x^*(B)$ for members of coalition Y. Note also, that $\frac{\partial p_x^*(B)}{\partial x} > 0$, $\frac{\partial p_x^*(B)}{\partial y} < 0$, and that $p_x^*(B) > p_x^*(B)$ if and only if $x > y$. Total conflict investment of an agent $i \in X$ can be calculated as:

$$E_i^*(B) = \frac{\sqrt{V\sqrt{x\,y}}}{1+\sqrt{\frac{y}{x}}}.$$

This expressions is strictly increasing in x as long as $x < y$ and becomes strictly decreasing for $x > y$. Using the derived solutions to calculate expected equilibrium payoff for an agent $i \in X$ as specified in Eq. (12) yields:

$$\pi_i(\mathbf{e}_\mathbf{i}^*, \mathbf{e}_{-\mathbf{i}}^*; B) = \frac{y(x - y - \sqrt{x\,y})}{(\sqrt{x}+\sqrt{y})^2}V,$$

which is strictly increasing in x. Note that this relation also implies that, for $i \in X$ and $j \in Y$, we have that $\pi_i(\mathbf{e}_\mathbf{i}^*, \mathbf{e}_{-\mathbf{i}}^*; B) > \pi_j(\mathbf{e}_\mathbf{j}^*, \mathbf{e}_{-\mathbf{j}}^*; B)$ if and only if $x > y$.

Conflict intensity in a complete bipartite network can be expressed as:

$$E^*(B) = \sqrt{(x\,y)^{\frac{3}{2}}V}. \qquad (14)$$

This expression is obviously increasing in x, as well as in y, the number of agents in each coalition. Solving the inequality $E_1^*(B) > E_2^*(B)$ yields the condition as stated in the proposition. ■

In a complete bipartite conflict network a coalition becomes more powerful if it has more members. The intuition for this result is similar to the star-shaped network: Assume for a moment that $x > y$, then proposition 5 implies that members of coalition X will in expectation win each bilateral conflict with higher probability, i.e., for $i \in X$ and $j \in Y$, we have that $p_{ij}^*(B) > p_{ji}^*(B) = 1 - p_{ij}$, and therefore $p_{ij}^*(B) > \frac{1}{2}$. Moreover, aggregated conflict investment $E_j^*(B)$ of an agent j of coalition Y is increasing in x but this increase is not sufficient to keep the win probabilities constant among all bilateral conflicts in which she is involved. This relation also holds with respect to equilibrium utility that is increasing in the number of members of the respective own coalition. If the lead in coalition membership is sufficiently large (out of the perspective of coalition X: if $x > y(3+\sqrt{5})/2 \approx 2.62\,y$) then it is possible that the conflict game results in positive equilibrium payoff for the members of the more numerous coalition.[12]

Conflict intensity is decreasing in the number of coalition members in complete bipartite networks. In this class of conflict structures peaceful resolution of conflicts is interpreted as resolving all conflictive links of at least one agent (because otherwise the resulting network is not a complete bipartite networks). Based on this interpretation peaceful conflict resolution in the class of complete bipartite conflict networks is beneficial.

[12] As s star-shaped network is a special case of a complete bipartite network, this result also holds for star-shaped conflict network, i.e., if the number of players in the periphery is 3 or larger, then agents at the periphery have positive equilibrium payoff.

3.4 Conflict Intensity and Centrality

The corollary that is presented below summarizes the results derived so far for the three types of considered conflict networks R, S, and B by establishing a relation between conflict intensity and network centrality, here eigenvector centrality, for the considered class of conflict networks. The following additional notation is used: The symmetric adjacency matrix G represents graph $\mathbf{g}$ and has elements g_{ij} where $g_{ii} = 0$ for all $i \in N$ (because no agent is in a conflictive relation with herself). The largest eigenvalue of G, denoted by $\mu(G)$, is real-valued and positive because G is symmetric. By the Perron-Frobenius theorem the components $(\mu_1(G), \ldots, \mu_n(G))$ of the eigenvector that corresponds to the largest eigenvalue $\mu(G)$ are all positive and frequently interpreted as a centrality measure of the respective nodes of graph $\mathbf{g}$.[13] Solving the characteristic equation for each network type implies that:

- for regular networks: $\mu(R) = d$ and $\mu_i(R) = 1$ for all $i \in N$.
- for complete bipartite networks: $\mu(B) = \sqrt{x\,y}$, and, assuming without loss of generality that $x > y$:

$$\begin{aligned} \mu_i(B) = &\quad 1 &&\text{for all } i \in X, \\ \mu_j(B) = &\quad \sqrt{\tfrac{x}{y}} &&\text{for all } j \in Y. \end{aligned}$$

- for star-shaped networks:[14] $\mu(B) = \sqrt{n-1}$, and

$$\begin{aligned} \mu_i(S) &= 1 \text{ for all } i \in N_c, \\ \mu_c(S) &= \sqrt{n-1}. \end{aligned} \tag{15}$$

Based on this notation the following relation holds:

Corollary 6 *For the class $\{R, S, B\}$ of conflict networks total conflict intensity is positively related to the number of conflicts in the respective network and negatively to the largest eigenvalue of its adjacency matrix. Individual conflict spending is, additionally, negatively related to individual eigenvector centrality.*

$$\text{For } \mathbf{g} \in \{R, S, B\}: \qquad E^*(\mathbf{g}) = \sum_{i \neq j} \frac{g_{ij}}{2} \sqrt{\frac{V}{\mu(G)}}, \qquad \textit{and} \tag{16}$$

$$e^*_{ij}(\mathbf{g}) = \frac{\mu_j(G)}{\mu_i(G) + \mu_j(G)} \sqrt{\frac{V}{\mu(G)}}. \tag{17}$$

[13] Here the importance of an agents depends on the number of links but also on the importance of her rivals. The google page rank algorithm is a frequently mentioned example in the context of this type of centrality measures.

[14] The results for complete bipartite networks can be used by setting, without loss of generality, $X = c$ and $Y = N_c$.

Proof. Applying the derived eigenvector results for the different network classes to the equilibrium solutions for each type of conflict network yields the statement in the corollary. ∎

This corollary facilitates the cross-comparison of conflict structures that belong to different types of conflict networks. Statements of the following type become possible: Assume that $\mathbf{g}, \mathbf{g'} \in \{R, S, B\}$ and that $\sum_{i \neq j} g_{ij} = \sum_{i \neq j} g'_{ij}$, i.e., both conflict networks have the same number of conflictive relations. Then the corollary implies that the conflict network with the smaller eigenvalue induces higher conflict intensity: if $\mu(G) > \mu(G')$ then $E^*(\mathbf{g}) < E^*(\mathbf{g'})$. Similarly, if two conflict networks $\mathbf{g}, \mathbf{g'} \in \{R, S, B\}$ have the same eigenvalue $\mu(G) = \mu(G')$ and $\sum_{i \neq j} g_{ij} > \sum_{i \neq j} g'_{ij}$ then the conflict intensity is higher in the conflict network that has a higher number of conflictive relations: $E^*(\mathbf{g}) < E^*(\mathbf{g'})$.

3.5 Peaceful Conflict Resolution

Within the considered classes of conflict networks total conflict intensity is positively related with the number of bilateral conflicts. From the perspective of peaceful conflict resolution this implies that it is always beneficial to resolve bilateral conflicts within each network class R, S, and B because overall conflict intensity is reduced. However, this does not imply that peaceful resolution of bilateral conflicts is always beneficial because the realtion between peaceful conflict resolution and conflict intensity only holds if the resulting network remains in the considered class, or can be compared according to corollary 6. In general this relation might not hold for conflict networks that are not part of those classes. We now provide an example where peaceful resolution of bilateral conflicts does in fact lead to an increase in total conflict intensity. Here, two centers of two identical star networks (that each have $n_c = n - 1$ agents in their respective periphery such that there are in total $2n$ agents in this conflict network) are in conflictive relation with each other. We are interested in the consequences for overall conflict intensity induced through resolving the central conflict between the center agents. The resulting graph of two stars with linked centers is denoted by S_2 and is represented, together with the situation after conflict resolution, in figure 3.

The payoff function of a center agent that is linked with the other center is:

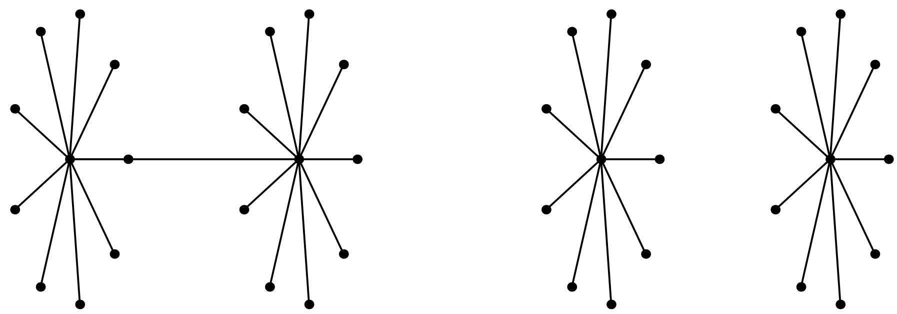
$$\pi_c(\mathbf{e_c}, \mathbf{e_{-c}}; S_2) = \sum_{i \in N_c} \frac{e_{ci}}{e_{ci} + e_{ic}} 2V + \frac{e_{cc}}{e_{cc} + e_{cc}} 2V - (\sum_{i \in N_c} e_{ci} + e_{cc})^2 - nV,$$

where e_{cc} denotes the conflict spending of one center against the other.[15]

Based on numerical solution techniques it is possible to calculate the conflict intensity $E^*(S_2)$ for this network constellation and to compare it with $2E^*(S)$, the analytical expression for conflict intensity in a network with two isolated

[15]Note that both star networks have the same number of agents in their periphery. It can be shown that the only interior equilibrium is symmetric in the sense that both center agent invest the same amount into the conflict against each other. Hence, it is not necessary to discriminate between the two center agents.

Figure 3: Example S_2 before (left) and after (right) peaceful conflict resolution



star-shaped conflict structures based on Eq. (11). Surprisingly, resolving the central conflict may actually imply an increase in conflict intensity. More precisely, if $n_c > 10$ then $E^*(S_2) < 2E^*(S)$, i.e., peaceful conflict resolution induces higher conflict intensity if each of the two stars has more than ten agents in the periphery.[16] If $n_c \leq 10$ then conflict intensity decreases.

The intuition for this result is based on the fact that the decrease in the number of conflictive relations among the center agents induces a negative externality on the respective rivals in the periphery. It can be shown that the aggregated conflict spending of the two centers is lower because they face less direct conflicts. However, they also shift part of the conflict investment from the resolved central conflict to the periphery. Agents in the periphery react to this increase in conflict spending of their rival (the respective center agent) by also increasing investment into this conflict. Hence, if the number of periphery agents is sufficiently large, those indirect effects (the externality induced by resolving the central conflict) by the periphery dominate the direct effect of decreased aggregated spending by the two center agents.

Hence, paying attention to the underlying structure of conflicts is crucial for the success of peaceful conflict resolution. The example also suggests that the resolution of bilateral conflicts should be targeted with respect to the underlying conflict structure to guarantee a reduction of conflict intensity. However, finding the bilateral conflict that would (by peacefully resolving it) induce the maximal decrease in conflict intensity requires an analytical solution of Eq. (18) which in general does not exist for conflict networks that go beyond the considered classes $\{R, S, B\}$. In the next section we derive partial results that indirectly characterize the bilateral conflict that induces the highest aggregated conflict spending.

[16] To apply those techniques it was assumed that $V = 1$. The result remains the same for different numerical values of V.

4 General Irregular Conflict Structures

The results derived in the previous section are based on the assumption that the conflict network belongs to one of the types R, S, or B. This assumption is now relaxed by extending the analysis to general irregular networks. As already mentioned, individual conflict spending can be charcaterized as the solution to the following system of $\sum_{i \in N} n_i$ first order equations:[17]

$$\frac{e^*_{ki}}{(e^*_{ik} + e^*_{ki})^2} V = E^*_i, \qquad \text{for all } k \in N_i \text{ and all } i \in N. \tag{18}$$

Analytical solutions for this system of equations do not exist because each equation is non-linear. However, the following reformulation yields a less complex system of equations and also allows to derive some additional results.

Combining the two first order conditions for two direct rivals i and j implies that in equilibrium:

$$\frac{e^*_{ij}}{e^*_{ji}} = \frac{E^*_j}{E^*_i}, \text{ and} \tag{19}$$

$$e^*_{ij} + e^*_{ji} = \frac{V}{E^*_i + E^*_j}. \tag{20}$$

This implies that the probability to win the bilateral conflict can be expressed in terms of aggregated conflict spending of the two rivals:

$$p(e^*_{ij}, e^*_{ji}) = \frac{E^*_j}{E^*_i + E^*_j} \tag{21}$$

Assume now that the agents are renamed in the order of the aggregated equilibrium investment that they spend in all their respective conflicts, i.e., $E^*_1 \leq E^*_2 \leq \ldots \leq E^*_n$. Then the following relation holds in equilibrium:

Proposition 7 *If agent i is in conflict with agent j and $i < j$ then agent i will win this conflict in expectation.*

Proof. If $i < j$ then $E^*_i \leq E^*_j$ because the agents are ordered. By Eq. (19) this implies that $e^*_{ij} \geq e^*_{ji}$. By the definition of the probability $p(e^*_{ij}, e^*_{ji}) \geq p(e^*_{ji}, e^*_{ij})$ and hence $p^*_{ij} \geq \frac{1}{2}$. ■

This result is counter-intuitive at first sight because it states that agent i will win the conflict against j although she invests in total less in all her conflicts than her rival j. However, the reason for the fact that j invests relatively more in total conflict spending is due to that fact that she either faces very aggressive rivals or because she has a lot of them. Both situations favor her direct rival i who can guarantee a high winning probability because agent j will not invest too

[17] The dependence of equilibrium conflict spendings on graph **g** is suppressed for notational convenience.

much into the conflict against i in comparison to her other conflicts.[18] Note, that the same intuition has been discussed in the section on star-shaped networks where the center agents is the one with highest total conflict investments but still looses in expectation in each bilateral conflict with the periphery.

By combining Eq. (19) and (20) it becomes obvious that individual conflict spending into a singular bilateral conflict is totally determined by the aggregated conflict spending of the two directly affected rivals:

$$e_{ij}^* = \frac{E_j^*}{(E_i^* + E_j^*)^2} V.$$

This allows the simplification of the system of $\sum_{i \in N} n_i$ first order conditions as specified in Eq. (18) to a system of equation with only n equations:

$$E_i^* = \sum_{j \in N_i} \frac{E_j^*}{(E_i^* + E_j^*)^2} V \text{ for all } i \in N.$$

However, this system of equations is still non-linear and can not be solved analytically. Overall conflict intensity can be indirectly characterized as:

$$E^* = \sum_{i \in N} E_i^* = \sum_{i \neq j} \frac{g_{ij}}{2} \frac{V}{E_i^* + E_j^*}.$$

Note that the same caveat as in footnote 18 applies here because E^*, as well as E_i^*, depend on the whole network structure $\mathbf{g}$. This implies that changes in the network structure $\mathbf{g}$ affect E^* in two ways: directly, because at least one g_{ij} is altered, but also indirectly through E_i^* of all $i \in N$ that also depend on $\mathbf{g}$.

Nevertheless, Eq. (20) can be used to determine the bilateral conflict that induces the highest aggregate conflict investment $E_{ij}^* = e_{ij}^* + e_{ji}^*$ because it will be inversely related to the sum of the aggregated conflict spending of the two affected agents i and j. This seems to suggest that isolated bilateral conflicts where affected agents do not have any additional rivals induce the highest levels of conflict spending. The question whether such an isolated conflict is the optimal target for peaceful conflict resolution must remain open because the feedback effects that are induced by resolving alternative (and more embedded) bilateral conflicts cannot be compared based on the partial results derived here.

5 Discussion

The model as presented here is based on a specific functional form. However, our results should also hold (at least qualitatively) for more general specifications. We assume, for instance, that the cost function is quadratic. Note that neither our existence proof nor the results with respect to the considered classes of

[18] A caveat of Proposition 7 is that the reordering of the agents depends on aggregated individual equilibrium spending which itself will obviously depend on the network structure.

conflict networks depend in a crucial way on this assumption.[19] Note also that a conflict game with a more general probability function, e.g., as in Skaperdas (1996) would be strategically equivalent to a model with a different (but convex) cost function.

We argued that in our set up heterogeneity among agents is implicitly induced by the relevant position in the potentially irregular network structure. Hence, adding another source of heterogeneity, for instance different valuations or different cost functions, might influence our results. An interesting extension of our model would relax the assumption that each agent can win (or loose) in principle the same amount of resources in each conflict. An alternative specification with respect to individual resource endowment would be the following: $V_i = V(n_i)$ with $\frac{\partial V_i}{\partial n_i} \leq 0$, for instance, $V_i = n_1^{-\alpha} V$ with $\alpha \in [0, 1]$ where $\alpha = 0$ is the case considered so far in the model. An interesting research question that could be addressed in this kind of extended model is whether the two sources of heterogeneity (resource endowment and network location) are of complementary nature or balance each other out.

6 Concluding Remarks

Analyzing conflict situations that are embedded in a network structure of conflictive relations yields constructive results with respect to equilibrium conflict intensity and network characteristics. While we confirm the intuitive statement that more singular conflicts imply higher conflict intensity for an important class of conflict structures, i.e., regular, star-shaped, and complete bipartite conflict networks, we also provide an example under which this statement is not true. Nevertheless, a general relation between conflict intensity and a prominent centrality measure is established for the considered class of conflict networks.

Extending the analysis to more general irregular networks is a complex issue due to the fact that no closed form equilibrium solutions exist. Partial results allow us at least to establish an inverse relation between aggregated conflict spending and individual conflict spending that also determines the win probability and the conflict intensity for each bilateral conflict. We leave a more extended analysis of this general case to further research.

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