Sharing Rules with Cooperative Production under the Threat of Sabotage [1]

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Abstract

This paper analyzes the consequences of a parametrized class of sharing rules on the propensity of individuals to sabotage each other in a cooperative production framework. The considered sharing rules differ in their sensitivity to relative input contributions which has an impact not only on equilibrium provision of individual labor (that increases the respective individual input contribution) but also on the propensity to sabotage others (which decreases the input contributions of sabotaged individuals). Necessary and sufficient conditions for an equilibrium where zero sabotage is the optimal equilibrium strategy are derived that depend on the sensitivity parameter of the sharing rule. It can be shown that sharing rules that are highly sensitive to differences in input contributions always induce an equilibrium with positive sabotage which can never be pareto-efficient. Sharing rules that instead put more emphasis on equal sharing lead to an equilibrium where zero sabotage is the optimal equilibrium strategy for all individuals.

Keywords: Sharing rules, sabotage, cooperative production

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1 Introduction

Modern compensation schemes frequently depend to a large extent on either individual (if observable) or relative (in comparison to others) performance measures, i.e. there seems to be a general tendency to put more emphasis on providing a significant part of the salary as variable compensation. This dependence of the individual salary on the respective performance is usually justified by incentive motivations, for instance, to reduce free-riding behavior. However, incentivizing agents through a highly sensitive performance based compensation scheme may result in detrimental consequences. In the context of a rank-order tournament model where effort is non-observable, Lazear (1989) argues that higher competitive pressure might induce agents to sabotage each other (which reduces the performance of others) instead of increasing their individual performance. The existence of this potentially destructive reaction to performance based compensation schemes will naturally affect the optimal design of those schemes. In Lazear (1989) the wage spread is the key parameter that can be controlled by the firm to influence the performance (and sabotage activity) of the agents. Under this assumption it is shown that wage compression might be profit increasing if agents have the option of sabotaging others. However, this assumption takes salaries as not being related to the economic performance of the whole firm. In fact, performance related compensation schemes frequently do not only depend on the relative individual performance but also on the overall performance of the firm, work group, etc. Examples are compensation schemes for CEO´s that partially consist out of stock options that depend on the share price (and therefore potentially also on the economic success) of the relevant firm, or research recognition programs where the individual bonus of a researcher does not only depend on the individual research record but also on the overall research performance of the department.

Those examples suggest that the competed ‘prize’ (e.g. salary increase, bonus etc.) might not be a fixed amount but also depend on the performance of the relevant agents. Therefore, it makes sense to interpret the competed prize as a jointly produced output. The problem of specifying a wage spread is then transformed to the specification of a sharing rule that determines how the jointly produced output is shared among the agents.[1]

Using a cooperative production model that incorporates a jointly owned production technology also enriches the analysis because the notion of pareto-efficiency makes sense in this context, i.e. the question can be raised whether a sharing rule exists for this model that yields a pareto-efficient allocation (output-labor-sabotage combination) as a non-cooperative equilibrium outcome.

The model that is introduced below is therefore based on a cooperative production set-up where individuals have to decide their respective amount of labor that determines their individual level of input. The vector of input contributions is assumed to be the only observable and contractable information.[2] The cooperative production technology transforms the provided total amount of input contributions into a single and divisible output. This output is shared among the individuals according to a parametrized sharing rule where the parameter determines the sensitivity of the sharing rule with respect to the respective input contributions of the individuals (with equal sharing and proportional sharing as polar cases). The model specification as described until now coincides with a standard cooperative production model, as it is introduced in Sen (1966), or Israelsen (1980).[3]

The non-standard element of the model is the additional strategic option of each individual to sabotage the labor contribution of others which will affect the input contribution of the sabotaged individual. From an individual strategic perspective a sabotaging agent decreases the input of the sabotaged individual which yields a higher relative input share (and therefore also relative output) for the sabotaging agent. At the same time exerting sabotage reduces total output which implies that positive sabotage is inefficient from a social welfare perspective because zero sabotage would (ceteris paribus) increase joint output. These two effects induce a

[1]Full competition for workers on the labor market would also imply that competing firms have to distribute the complete produced surplus to the workers, see Lazear (1989).

[2]This is the basic difference to the literature on team compensation (e.g. Holmstrom 1982) because there inputs and effort levels are not observable. The resulting problems of moral hazard do not exist in the cooperative production set-up as there is no incomplete information with respect to input contributions.

[3]An important result of Sen (1966) is the specification of a sharing rule that induces a pareto-efficient allocation in this framework. A natural question would be if this result can also be sustained in a model where sabotage is an option.

strategic trade-off for each individual that is highly dependent on the sensitivity of the sharing rule with respect to the input contributions of the individuals. Therefore, the individual decision whether to exert sabotage or not depends in particular on the specification of the sharing rule. As the sharing rule is the same for all individuals, it is possible to derive general conditions, i.e. restrictions on the design of the sharing rule, that guarantee the existence of an equilibrium where all individuals refrain from sabotage.

The underlying theoretical model is formulated in rather general terms that allows alternative interpretations of the relevant framework. For instance, if the set of individuals is interpreted as an economy where the sharing rule governs the distribution of the produced total surplus then the parametrized sharing rule can be interpreted as the level of tax-based redistribution within this economy. In this case equal sharing would coincide with a 100-percent tax and full redistribution while proportional sharing could be interpreted as no redistribution/taxes at all with an individual wage that solely depends on the relative individual provision of inputs.[4] The underlying question is then: how much redistribution is necessary such that individuals refrain from sabotaging others?

In the next section the model is formally introduced and a necessary condition for an equilibrium with zero sabotage is derived. For this kind of equilibrium it is possible to specify a sharing rule that induces a pareto-efficient equilibrium outcome where individuals (voluntarily) do not exercise their sabotage option. Unfortunately, a direct existence proof is not available for this kind of set-up. Therefore a detour is taken by first assuming that sabotage is exogenously constrained to be equal to zero and by deriving the unique equilibrium for this kind of restricted set-up. The resulting allocation is then a candidate for an equilibrium in the unrestricted model with potential sabotage. Transforming this candidate solution into a full-fledged equilibrium for the unrestricted model yields an additional sufficient condition that guarantees that the candidate solution is in fact an equilibrium where zero sabotage is the optimal response of each individual. This sufficient condition is stricter than the necessary condition and does not cover the specification of the sharing rule that induces the efficient allocation in a model without sabotage (Sen 1966).[5] A final section concludes by addressing potential

[4] I thank Jenny de Freitas for suggesting this interpretation.

[5] Therefore, there might exist cases where the rule suggested by Sen yields positive sabotage

limitations and further research possibilities.

Besides Lazear (1989) there exists a limited but growing literature on sabotage in different competitive situations: Konrad (2000) analyzes sabotage in a contest game with homogenous agents, derives a necessary condition for the existence of an equilibrium without sabotage, and shows that this condition is more strict if the number of contestants is increasing.[6] The homogeneity assumption is relaxed in Amegashie and Runkel (2007) for a two-stage all pay-auction with four heterogeneous players where the sabotage decision is made before the auction is played. The results are highly dependent on how the players are matched against each other in the different stages. Chen (2003) introduces sabotage in a rank-order tournament model as in Lazear (1989). There, players that are highly able are more sabotaged than the less able ones which leads to double inefficient outcomes because the highly able players may loose the tournament. Münster (2007) shows that these results are due to the specification of the cost function. However, all papers mentioned so far assume that the valuation of the prize is exogenously determined and does not depend on the exerted productive effort or labor. A model that takes this interdependence into account and that is more related is the sabotage game that is introduced in Beviá and Corchón (2006). They derive necessary and sufficient conditions for an equilibrium with zero sabotage as optimal strategy based on a cooperative production set-up with heterogeneous players and a generalized class of sharing rules. However, in their model the agents are forced to distribute their entire fixed labor time either to productive labor or to destructive sabotage activity, i.e. agents can only control the proportion of labor versus sabotage that they contribute. This assumption is relaxed in the model presented below by separating the individual decision of exerting labor and/or sabotage and additionally by inducing a disutility on each activity.[7] Another difference is the assumption of homogeneity of the players and a specific (parametrized) sharing rule that allows to address the question whether an efficient equilibrium can be sustained under the potential threat of sabotage.

which immediately implies that the allocation is not pareto-efficient.

[6]The model presented below can be interpreted as a generalization of this set-up by allowing non-linear cost functions, a non-constant prize that depends on the contributions of the contestants, and a more general contest success function that also incorporates mixtures between the usual (proportional) contest success function and equal sharing.

[7]This specification implies that there exists an implicit third alternative, which is leisure.

2 The Model

There is a set $N = \{1, \ldots, n\}$ of individuals that voluntary provide labor to a cooperative production process, i.e. each individual $i \in N$ has to decide which non-negative amount of labor $l_i \in \Re_+$ to provide to production. The vector of labor input of all individuals is denoted by $\mathbf{l} = (l_1, \ldots, l_n)$ and its sum by $L = \sum_{j \in N} l_j$. Additionally, individual i has the possibility to sabotage individual j with intensity $s_{ij} \in \Re_+$. A strategy of individual i is described by the vector $(l_i, \mathbf{s_i})$ where $\mathbf{s_i} = \{s_{ij}\}_{j \neq i}$ is the vector of sabotage of individual i against all other individuals. The sabotage that individual i suffers from by all other individuals is denoted as $\mathbf{s^i} = \{s_{ji}\}_{j \neq i}$. The set of strategies by all players is $(\mathbf{l}, \mathbf{s}) = (l_1, \ldots, l_n, \mathbf{s_1}, \ldots, \mathbf{s_n})$. The amount of labor l_i of individual i and the sabotage $\mathbf{s^i}$ it suffers determines its effective input $r^i = r(l_i, \mathbf{s^i})$ which is increasing in own labor and decreasing in each element $s_{ji} \in \mathbf{s^i}$. As in Konrad (2000) it is assumed that the level of effective input of individual i, if not sabotaged at all, is equal to individual labor: $r(l_i, 0, \ldots, 0) = l_i$. The sum of effective input $R = \sum_{j \in N} r_i$ is transformed into a single divisible output with a concave production technology $f(R) = R^\epsilon$ with constant elasticity $\epsilon \in (0, 1]$ as in Sen (1966). The produced output is then distributed among the individuals and consumed. The individual consumption of individual $i \in N$ is denoted by x_i while the vector of consumption is $\mathbf{x} = (x_1, \ldots, x_n)$. Feasibility requires that the sum of individual consumption cannot exceed total production: $\sum_{j \in N} x_j \leq f(R)$.

Preferences are represented by continuous utility functions which are increasing and concave in consumption, and decreasing and concave in individual labor contribution and sabotage activity.[8] In line with the two mentioned studies it is assumed that all individuals are homogenous with respect to their preferences. The characteristics of the utility function of individual i can be stated formally in the following way:

$$u^i = u(x_i, l_i, \mathbf{s_i}) \text{ with } u^i_x > 0, u^i_{xx} \leq 0, u^i_l \leq 0, u^i_{ll} < 0, u^i_s \leq 0, u^i_{ss} < 0, \quad (1)$$

where $u^i_s = \frac{\partial u^i}{\partial s_{ij}}$ for all $j \neq i$.

[8] The disutility induced by exerting labor and sabotage would coincide with a cost function that is convex in labor and sabotage activity and additively separable with respect to consumption.

The set-up as specified until now can be interpreted as an allocation problem. A feasible allocation for this framework is then defined as a tuple $(\mathbf{x}, \mathbf{l}, \mathbf{s})$ of consumption, labor, and sabotage activities for each individual, where feasibility requires that $\sum_{i \in N} x_i = f(R)$. Also, a pareto-efficient allocation can be defined in the usual way: a feasible allocation $(\mathbf{x}, \mathbf{l}, \mathbf{s})$ is efficient if there exists no other feasible allocation $(\mathbf{x}', \mathbf{l}', \mathbf{s}')$ such that $u(x'_j, l'_j, \mathbf{s'_j}) \geq u(x_j, l_j, \mathbf{s_j})$ for all $j \in N$ and $u(x'_j, l'_j, \mathbf{s'_j}) > u(x_j, l_j, \mathbf{s_j})$ for at least one $j \in N$. A feasible allocation that is also pareto-efficient is denoted by $(\mathbf{x^e}, \mathbf{l^e}, \mathbf{s^e})$.

The characterization of a pareto-efficient allocation is a normative question that would require that individuals can be forced to exert specific levels of labor and sabotage. It does not explain how this allocation can be generated if individuals act strategically, i.e. exert labor and sabotage voluntarily. For this kind of strategic analysis a sharing rule has to specified that describes ex-ante how the jointly produced output is shared among the individuals. The following parametrized sharing rule, introduced in Sen (1966), is a convex combination of equal sharing and proportional sharing based on the individual effective input. This sharing rule has the following functional form:

$$sh^i(\mathbf{l}, \mathbf{s}) = \left(\alpha \frac{r^i}{R} + (1 - \alpha) \frac{1}{n} \right) R^\epsilon,$$

where the parameter $\alpha \in [0, 1]$ is the weight on the proportional share (i.e. the relative input contribution) in the sharing rule.[9] For the alternative interpretation of this set-up as a tax-based redistribution framework the parameter α can be interpreted as the degree of redistribution or taxation: a low α would coincide with a relatively high tax level that yields a relative high tax revenue that is redistributed to the full extent as an equal lump-sum bonus to all individuals. Setting $\alpha = 1$ instead would imply that the produced output is distributed according to relative input contributions without any taxation or redistribution.

The utility function for individual i in this strategic framework can then be expressed as $u(sh^i(\mathbf{l}, \mathbf{s}), l_i, \mathbf{s_i})$ where consumption x_i is here substituted by the respective share $sh^i(\mathbf{l}, \mathbf{s})$.

The following technical assumption implies that exerting a small positive amount

[9]For instance, $\alpha = 1$ would yield the proportional sharing rule, while $\alpha = 0$ would imply equal sharing among all individuals.

of initial labor and/or sabotage does not induce disutility which makes the restrictions on the utility function in Equation (1) slightly more specific.

Assumption 1

$$u_l^i = 0 \textit{ for } l_i = 0 \quad \textit{while} \quad u_l^i < 0 \textit{ for } l_i > 0,$$
$$u_s^i = 0 \textit{ for } s_{ij} = 0 \quad \textit{while} \quad u_s^i < 0 \textit{ for } s_{ij} > 0.$$

This assumption rules out equilibria at the boundary where individuals would prefer to not exert labor at all.[10]

Deriving properties of an equilibrium where zero sabotage is the optimal strategy by all players requires a characterization of this kind of equilibria. The following lemma states that such an equilibrium has to be symmetric. This result is then used to derive a necessary condition for an equilibrium with zero sabotage as optimal strategy.

Lemma 1 *An equilibrium with zero sabotage,* $\mathbf{s}^* = (0, \dots, 0)$*, as optimal strategy must be symmetric, i.e. all players will contribute identical levels of labor:* $l_i^* = l_j^*$ *for all* $i \neq j$*.*

Proof: The proof proceeds by contradiction based on the following steps:

Claim 1: There is no free-riding in an equilibrium where zero sabotage is the rational strategy by all players, i.e. each individual will exert positive labor $l_i^* > 0$*.*

This is the direct consequence of Assumption 1 because exerting a marginal amount ε of labor does not induce any disutility:

$$u(sh^i(\varepsilon, \mathbf{l}^*_{-\mathbf{i}}, \mathbf{s}^*), \varepsilon, \mathbf{s}^*_{\mathbf{i}}) > u(sh^i(0, \mathbf{l}^*_{-\mathbf{i}}, \mathbf{s}^*), 0, \mathbf{s}^*_{\mathbf{i}}).$$

This last inequality rules out equilibria at the boundary which implies that the equilibrium level of labor is determined by the first-order condition which is expressed as:

$$\frac{\partial u^i}{\partial l_i} = u_x^i \; sh_l^i + u_l^i = 0 \tag{2}$$

[10] The assumptions on the utility function are satisfied, for example, by the following specification: $u^i = sh^i(l, s) - (l_i^2 + \sum_{j \neq i} s_{ij}^2)$.

Claim 2: $sh_l^i > 0$ for $l_i > 0$.

This inequality holds because $sh_l^i = [\alpha(L - l_i) + \epsilon(\alpha\, l_i + (1-\alpha)L/n)]L^{\epsilon-2} > 0$ by observing the fact that all terms in this summation are positive (with at least one term strictly positive).

Claim 3: If $l_i > l_k$ *then* $sh_l^i < sh_l^k$.

The claim follows by using Claim 2, the fact that $(\epsilon - 1) < 0$, and by using the following reformulation: $sh_l^i = \alpha\, l_i\, L^{\epsilon-2}(\epsilon - 1) + L^{\epsilon-1}(\alpha + \epsilon(1-\alpha)/n)$.

It is now shown that an equilibrium without sabotage and asymmetric labor contributions yields a logical contradiction. Assume that there are two players i and k that exert different levels of labor in equilibrium: $l_i > l_k$. By the established claims and Equation (2) it can be shown that: $0 = \frac{\partial u^i}{\partial l_i} < \frac{\partial u^k}{\partial l_k} = 0$, a contradiction. The statement of the lemma follows directly. □

Symmetry alone does not allow to make statements about existence of equilibria. However, the symmetry result can be used to establish the following necessary condition for an equilibrium where all players voluntarily do not exert any sabotage at all.

Proposition 1 *In an equilibrium where zero sabotage is the equilibrium strategy of all players, the weight on proportional sharing is lower than the elasticity of production:* $\alpha \leq \epsilon$.

Proof: By Lemma 1 an equilibrium with zero sabotage is symmetric, i.e. $l_i^* = l_k^*$ for all $i \neq k$. The fact that no player exerts positive sabotage in equilibrium can be expressed as the following inequality $\frac{\partial u(sh^i(l^*,0),l_i^*,0)}{\partial s_{ij}} = u_x^i sh_s^i + u_s^i \leq 0$. This inequality must be satisfied which is the case if (using the fact that $u_s^i = 0$ for $s_{ij} = 0$ by Assumption 1):

$$sh_s^i = r_s^i\,(L^*)^{\epsilon-2}[-\alpha\, l_i^* + \epsilon(\alpha\, l_i^* + L^*(1-\alpha)/n] \leq 0.$$

The symmetry result of Lemma 1 implies that $nl_i^* = L^*$. Using this simplification it can be shown that the last inequality is satisfied if $\alpha \leq \epsilon$. □

The necessary condition bounds the weight on proportional share with respect to the elasticity of production. This makes intuitive sense as the higher the elasticity of production the more impact has the individual input contribution on produced output. Therefore, in a set-up with a highly elastic production function sabotage is more costly because it reduces inputs while labor becomes more efficient with respect to production. Both effects imply that individuals prefer to exert productive labor instead of unproductive sabotage, i.e. an equilibrium with zero sabotage as optimal response can be sustained under a sharing rule that puts more weight on proportional sharing (up to the elasticity of production).

Note that the necessary condition also implies that a sharing rule that puts more weight on proportional sharing than elasticity of production (i.e. $\alpha > \epsilon$) will always induce positive sabotage in equilibrium. The following statement characterizes pareto-efficient allocations and shows that positive sabotage activity violates pareto-efficiency. Hence, a sharing rule that is too sensitive to relative input contributions (i.e. where $\alpha > \epsilon$) cannot yield a pareto-efficient equilibrium allocation.

Lemma 2 *A pareto-efficient allocation* $(\mathbf{x^e}, \mathbf{l^e}, \mathbf{s^e})$ *cannot entail positive sabotage, i.e.* $\mathbf{s^e} = (0, \ldots, 0)$*, and must satisfy the following* $n+1$ *equations:*

$$f'(L^e) = -\frac{u_l^i}{u_x^i} \text{ for all } i \in N, \tag{3}$$

$$\sum_{i \in N} x_i^e = f(L^e). \tag{4}$$

Proof:

Claim 1: A pareto-efficient allocation cannot entail positive sabotage.

The proof of the claim follows by contradiction. Suppose that an allocation is efficient where at least one individual $j \in N$ exerts positive sabotage. Denote this allocation as $(\mathbf{x}', \mathbf{l}', \mathbf{s}')$ where for at least one $k \neq j$ $s'_{jk} > 0$. However, this allocation is strictly dominated by an allocation where individual j refrains from sabotage and the additional produced output is, for instance, equally distributed to all individuals. Formally such an allocation can be expressed as $(\hat{\mathbf{x}}, \mathbf{l}', \hat{\mathbf{s}})$ where $\hat{\mathbf{s}} = (\hat{\mathbf{s}}_\mathbf{j} = (0, \ldots, 0), \mathbf{s}'_{-\mathbf{j}})$ and $\hat{x}_i = x'_i + [f(\hat{R}) - f(R')]/n > x'_i$. The last inequality follows from the fact that $\hat{R} = \sum_{i \in N} r(l'_i, \hat{\mathbf{s}}^\mathbf{i}) > \sum_{i \in N} r(l'_i, \mathbf{s}'^\mathbf{i}) = R'$ because $r(l'_k, \hat{\mathbf{s}}^\mathbf{k}) > r(l'_k, \mathbf{s}'^\mathbf{k})$ and $r(l'_i, \hat{\mathbf{s}}^\mathbf{i}) = r(l'_i, \mathbf{s}'^\mathbf{i})$ for all $i \neq k$. Note that feasibility still holds because $\sum_{i \in N} \hat{x}_i = f(\hat{R})$.

This implies that $u(\hat{\mathbf{x}}_\mathbf{i}, \mathbf{l}'_\mathbf{i}, \hat{\mathbf{s}}_\mathbf{i}) > u(\mathbf{x}'_\mathbf{i}, \mathbf{l}', \mathbf{s}'_\mathbf{i})$ for all $i \neq j$ because $\hat{x}_i > x'_i$ and $\hat{\mathbf{s}}_\mathbf{i} = \mathbf{s}'_\mathbf{i}$

but also that $u(\mathbf{\hat{x}_j}, \mathbf{l'_j}, \mathbf{\hat{s}_j}) > u(\mathbf{x'_j}, \mathbf{l'_j}, \mathbf{s'_j})$ because $\hat{x}_j > x'_j$ and $\hat{s}_{jk} = 0 < s'_{jk}$. Hence, an allocation $(\mathbf{x'}, \mathbf{l'}, \mathbf{s'})$ where some individual exerts positive sabotage cannot be pareto-efficient because $(\mathbf{\hat{x}}, \mathbf{l'}, \mathbf{\hat{s}})$ is preferred by all individuals.

Claim 2: A pareto-efficient allocation $(\mathbf{x^e}, \mathbf{l^e})$ *must satisfy Equation (3) and (4).*

A pareto-efficient allocation in this set-up can be expressed as the set of feasible allocations that maximizes the following weighted linear social welfare function $\sum_{i \in N} \lambda_i u(x_i, l_i)$ where $\lambda_i > 0$ for all $i \in N$ (by Negishi´s theorem, see Proposition 16.E.1 and 16.E.2 in Mas-Colell and Green 1995). The first order conditions of an (interior) solution to this maximization problem can be formulated as in Equation (3) and (4). As the maximization problem is well behaved (positive sabotage is ruled out by Claim 1), first order conditions are also sufficient.□

However, even if exerting no sabotage is an equilibrium strategy, the resulting allocations might be non-efficient in this framework, for instance, because of over- or underproduction. In those cases players are incentivized too much or not enough in comparison with an allocation that maximizes social surplus (Equation (3) would be violated). In Sen (1966) it is shown that in a framework where players do not have the possibility to sabotage others, the optimal weight on proportional sharing which does neither lead to over- nor to underproduction in equilibrium is $\alpha^* = \epsilon$. Hence, Proposition 1 implies that if an equilibrium with zero sabotage as equilibrium strategy does in fact exist then the result of Sen could also be sustained in a framework with potential sabotage, i.e. the framework of Sen would be "sabotage-proof". This is summarized in the following proposition:

Proposition 2 *In an equilibrium where zero sabotage is the equilibrium strategy of all players, the weight* $\alpha^* = \epsilon$ *on proportional sharing induces a pareto-efficient allocation as equilibrium outcome.*

Proof: By lemma 1 an equilibrium without sabotage must be symmetric. Rewriting the first order condition as stated in Equation (2) yields:

$$f'(L)\left[\alpha\frac{n-1}{n\epsilon} + \frac{1}{n}\right] = \frac{u_l^i}{u_x^i}. \tag{5}$$

Note that specifying $\alpha = \epsilon$ implies that Equation (5) and Equation (3) coincide. Equation (4) is satisfied trivially by the specification of the share function.□

The last proposition was based on the assumption that an equilibrium with zero sabotage as optimal strategy does in fact exist for $\alpha = \epsilon$. However, Proposition 1 is only a necessary condition for this kind of equilibrium. As usual existence proofs are not available for this kind of model,[11] the following detour is taken to prove existence: a restricted model is introduced where sabotage is exogenously constrained to be equal to zero. For this set-up a constructive existence result and the respective equilibrium allocation is provided. The derived equilibrium allocation is then taken as a candidate for the non-restricted model where sabotage is a potential option.

The following lemma provides the existence result for the restricted model.

Lemma 3 *There exists a unique equilibrium labor allocation in a set-up where sabotage is exogenously prohibited.*

Proof: The restricted model where sabotage is exogenously prohibited coincides with the set-up that is used in Lemma 1. Therefore, the equilibrium must be interior, i.e. can be characterized by Equation (2) which can also be expressed as:

$$\alpha(\epsilon - 1)l_i L^{\epsilon-2} + \alpha L^{\epsilon-1} + \epsilon L^{\epsilon-1}(1-\alpha)/n = -u_l^i/u_x^i. \tag{6}$$

Note that the right-hand side of Equation (6) is zero for $l_i = 0$ because of Assumption 1 and increasing because $\frac{\partial(-u_l^i/u_x^i)}{\partial l_i} = \frac{u_l^i u_{xx}^i - u_{ll}^i u_x^i}{(u_x^i)^2}$ which is positive by the assumptions on the utility function. The left-hand side of Equation (6) is positive for $l_i = 0$ by Claim 2 and decreasing because $sh_{ll}^i = \alpha(\epsilon-1)L^{\epsilon-3}(2(L-l_i) + \epsilon l_i) + \epsilon(\epsilon-1)L^{\epsilon-2}(1-\alpha)/n$ is negative (both terms in this equation are negative by inspection). The derived properties imply that there must exist a unique level of labor that equalizes right- and left-hand side of Equation 6 which establishes existence and uniqueness.□

Based on Lemma 1 and Equation (2), the symmetric equilibrium level of labor l^* is implicitly (as the marginal utilities also depend on l) defined as the solution to the following equation:

$$l = \left(-\frac{u_x^i}{u_l^i}\frac{\alpha(n-1)+\epsilon}{n^{2-\epsilon}}\right)^{1/(1-\epsilon)},$$

[11] The utility function may not be concave, the model is neither a supermodular nor an aggregative game.

which exists by Lemma 3. This expression will be the candidate solution for the non-restricted model where sabotage is not prohibited anymore.

The equilibrium in the restricted model is now interpreted as the candidate equilibrium for the non-restricted model where sabotaging others is a potential option. This candidate is in fact a full-fledged equilibrium in the non-restricted set-up if it can be sustained ever under the additional option of potential sabotage. Therefore it remains to be shown that for each individual $i \in N$ there exists no profitable individual deviation from the candidate allocation $(l_i^* = l^*, \mathbf{s_i^*} = (0, \ldots, 0))$. Checking all possible individual deviations yields a still missing sufficient condition for an equilibrium where zero sabotage is the optimal equilibrium strategy which completes the equilibrium analysis.

Proposition 3 *If the weight on proportional sharing is sufficiently low* $(\alpha \leq \frac{\epsilon}{n(1-\epsilon)+\epsilon})$ *then an equilibrium exists that entails zero sabotage as optimal strategy in the sabotage game.*

Proof: For each individual $i \in N$ there are three possible deviations from the equilibrium candidate allocation $(l^*, \mathbf{s_i^*})$ that have to be excluded as being profitable:

1. Deviation: $(l_i \neq l^*, \mathbf{s_i^*})$.
   A deviation of this type cannot be profitable because this kind of deviation would still be in the framework of Proposition 3 where it was proved that $(l^*, \mathbf{s_i^*})$ is the unique equilibrium. Hence, $l_i \neq l^*$ cannot be a profitable deviation.

2. Deviation: $(l_i^*, s_{ij} > 0)$.
   This kind of deviation will not be profitable if it can be shown that $\frac{\partial u(sh^i(l^*, s_i, s^*_{-i}), l_i^*, s_i)}{\partial s_{ij}} \leq 0$. This inequality can also be expressed as:
   $$\alpha(\epsilon - 1)l_i^* + (1-\alpha)\epsilon R/n \geq \frac{-u_s^i}{u_x^i}\frac{R^{2-\epsilon}}{r_s^j} \tag{7}$$
   Irrespective of the chosen level of sabotage of player i, there exists a lower bound on total output for this kind of deviation as player i does not suffer sabotage by anybody, i.e. $(\mathbf{s^i})^* = (0, \ldots, 0)$. This implies that $R \geq l_i^*$ for

all levels of sabotage that player i chooses. Using this lower bound and the fact that the right-hand side of Equation (7) is negative, it can be shown that the left-hand side of Equation (7) is positive if $\alpha \leq \frac{\epsilon}{n(1-\epsilon)+\epsilon}$. In this case Equation (7) is always satisfied.

3. Deviation: $(l_i \neq l_i^*, s_{ij} > 0)$.
The same argumentation as before holds for this type of deviation. Equation (7) is changed slightly to: $\alpha(\epsilon-1)l_i + (1-\alpha)\epsilon R/n \geq \frac{-u_s^i}{u_x^i}\frac{R^{2-\epsilon}}{r_s^j}$, and the lower bound on total output is now $R \geq l_i$ for all levels of sabotage of player i. Hence, the same condition that was derived before also holds in this case, i.e. the deviation is not profitable if $\alpha \leq \frac{\epsilon}{n(1-\epsilon)+\epsilon}$. $\square$

The condition $\alpha < \frac{\epsilon}{n(1-\epsilon)+\epsilon}$ guarantees that an equilibrium with zero sabotage as optimal strategy does always exist. Hence, this condition is a sufficient condition for an equilibrium with zero sabotage. Note also, that the necessary condition for an equilibrium with zero sabotage ($\alpha \leq \epsilon$) is weaker because $\frac{\epsilon}{n(1-\epsilon)+\epsilon} \leq \epsilon$. The last inequality is strict in general (the only exception is $\epsilon = 1$ which is the case if the production function is homogeneous of degree one).[12] This implies that the weight on proportional sharing plays a crucial role in determining whether an equilibrium that entails zero sabotage as optimal strategy exists: If the weight on proportional sharing is sufficiently low ($\alpha < \frac{\epsilon}{n(1-\epsilon)+\epsilon}$) then such an equilibrium does exist because the increase of the relative input share does not translate to a large increase in relative output. However, if the weight on proportional sharing is higher than the elasticity of production ($\alpha > \epsilon$), then all individuals will sabotage others in equilibrium.[13]

[12] In this case the necessary and sufficient condition coincide and the proportional sharing rule ($\alpha = 1$) induces an efficient allocation in equilibrium, a result that is also derived in Fabella (1988). The reason for this result is that a linear production function does not induce any individual externality for this kind of set-up.

[13] For intermediate cases, where $\frac{\epsilon}{n(1-\epsilon)+\epsilon} < \alpha < \epsilon$, it can be shown that an interior equilibrium where all individuals exert positive level of sabotage does not exist. However, for asymmetric equilibria (where only some individuals exert positive sabotage while others exert none) such a non-existence result could not be provided and also a numerical example of a specific utility function that would result in such an equilibrium could not be found. This seems to suggest that the sufficient condition is rather strong. A weaker sufficient condition that depends on the functional form of the utility function (derived from Equation (7) by using the same

The intuition for the last mentioned statement is also straight forward from an individual perspective: Here, the loss in production that results from sabotage activity (and the additional individual cost from sabotage) is more than compensated by the relatively high weight on the proportional sharing that rewards the individual input contribution more than equal sharing. However, all individuals replicate this strategic behavior which induces an inefficient outcome because the same outcome could have produced with less input and zero sabotage.

Note also that the upper limit on the weight of proportional sharing, $\frac{\epsilon}{n(1-\epsilon)+\epsilon}$, is decreasing in the number of individuals. In other words the propensity to sabotage others is higher if there are more individuals involved. This is quite intuitive: from the specification of the utility function it is clear that the efficiency of sabotage is decreasing in its amount. This implies that low levels of sabotage directed against a lot of opponents are more efficient than higher levels of sabotage directed against few opponents, i.e. the propensity to sabotage others is increased if there are a lot of opponents. Hence, the sufficient condition must be more strict for higher numbers of individuals to still guarantee zero sabotage in equilibrium.

3 Concluding Remarks

Modern compensation schemes that depend on relative performance could be interpreted as a parametrized sharing rule in a cooperative production set-up where the jointly produced output is partially shared depending on the relative input contribution of the respective individuals. However, the underlying incentive motivation for the establishment of such a scheme could have adverse consequences because individuals might sabotage others as an optimal response to the increased competitive pressure that is created by such a compensation scheme. This kind of intuition is verified by the model presented here: An equilibrium that entails zero sabotage as optimal response by all individuals does only exist if the sharing rule puts sufficient weight on equal sharing of the jointly produced output (which would coincide with the fixed payment part of the compensation scheme).

lower bound on output) is $-\frac{u_s^i}{u_l^i r_s^j} > \frac{(n-1)(1-\epsilon)}{n^\epsilon}$. The fact that this condition depends on the functional form of the utility function makes general statements with respect to this inequality difficult.

If the sharing rule puts a lot of weight on proportional sharing (i.e. variable payment with respect to relative performance) then the optimal strategy for each individual is to engage partly in sabotage activity. The important limit parameter is here elasticity of production which implies that the key result of the paper is robust with respect to the specific functional form of the utility function (as long as it is inside the family specified above). However, this would not be the case if individuals would be heterogeneous with respect to their preferences which might be an interesting further research possibility. A caveat for this potential generalization is the fact that pareto-efficient allocation in this framework can only be implemented for specific production functions and only with specific sharing rules that are not contained in the class considered here (Beviá and Corchón 2007). A second best approach with respect to the efficiency of specific sharing rules could be a promising future research possibility for this kind of generalized framework.

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